

DISTORTION DISTRIBUTION AND CONVERGENCE ANALYSIS OF SPHERICAL DIAMOND DISCRETE GRIDS

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ABSTRACT:

Four kinds of subdivision methods of Global Discrete Grid are presented in this paper. Furthermore, we systematically and comprehensively compare and analyze the distortion distribution and convergence of spherical diamond discrete grids. Especially, the Electrical Potential Energy is introduced as index to measure the distortion distribution of spherical diamond discrete grids. The result shows that the properties including the distortion of the area of the grid cell and the distortion of the angle of the grid cell or from the global uniformity of spherical diamond discrete grids which are obtained by recursively bisecting the great arcs on the surface of the globe starting with the icosahedron are much better than the properties of the other three discrete grid obtained by subdivision methods.

1. INTRODUCTION

Spherical diamond discrete grids are more easily to search neighboring cells and build hierarchical relation, and more suitable to manage global distributed massive spatial data and update local data dynamically, since their geometric structure is similar to the structure of square grids on planar, their coding method is simpler than that of triangular grid or hexagon grid, and they are oriental uniform and radially symmetrical. According to the basic polyhedron (such as, octahedron, icosahedron) and recursive bisection method, foreign and domestic researchers have proposed several different methods to construct spherical rhomboidal discrete grids which are all applied to manage and integrate the global multi-scale and multi-resolution spatial data like great arc bisecting method, longitude and latitude bisecting method, mixed bisecting method based on octahedron, and great arc bisecting method based on icosahedron. The applications based on octahedron include a hierarchical spatial data structure for global geographic information systems (Goodchild, 1922), continuous indexing of hierarchical subdivisions of the globe (Barthildij, 2001), spatial data quality and hierarchical comprehensive data model (Dutton, 1999; Gold, 2000; Li, 2003). The applications based on icosahedron are spherical hierarchical data index (Fekete, 1990), global navigation model (Lee, 2000), geodesic discrete global grid system (2003, Sahr).

Getting spherical discrete grid, the geometrical characteristics of which is excellent and distortion distribution of which is uniform, is major subject of the research of Digital Discrete Grid (Goodchild, 2000). But from spherical geometry we know there is a method by which we can get nearly the same instead of absolute the same geometrical characteristics (such as, area, length, and angle) as the planar raster. Before making full use of spherical diamond discrete grids, we need to figure out the questions, such as what's the similarity, what's the regulation of distribution of the area and angle, and are they converge, and analysis the numerical error of the grid model.

2. GENERATION METHOD OF SPHERICAL DIAMOND DISCRETE GRIDS

In this paper, we will introduce four kinds of methods by which spherical diamond discrete grids are generated. The grids include spherical diamond discrete grids which are obtained by recursively bisecting the great arcs on the surface of the globe starting with the octahedron, spherical diamond discrete grids which are obtained by recursively bisecting lonlat on the surface of the globe starting with the octahedron, spherical diamond discrete grids which are obtained by mixed method on the surface of the globe starting with the icosahedron and spherical diamond discrete grids which are obtained by recursively bisecting the great arcs on the surface of the globe starting with the icosahedron. They are obtained by projecting regular solids to the surface of sphere. The spherical diamond is combined by two triangles which are generated by bisecting different lines (the great arc, geodesic line or mixed lines) on the surface. Due to the different regular solids and bisecting methods, the geometrical characteristics and distortion convergence of the discrete grids are different. Next we will analyze the characteristics of the spherical diamond discrete grid.

3. GEOMETRICAL CHARACTERISTICS OF SPHERICAL DIAMOND DISCRETE GRID

At present, statistical analysis of geometry characteristics of grid cell is the major method used to research the geometry characteristics of the spherical discrete grids. Through analyzing the area of grid cell and major-minor axis ratio of the four kinds of spherical diamond discrete grids by statistical methods, the paper obtains the following conclusions.

- ① The distortion characteristics of the angle and area of the spherical diamond discrete grids which are obtained by recursively bisecting the great arcs on the

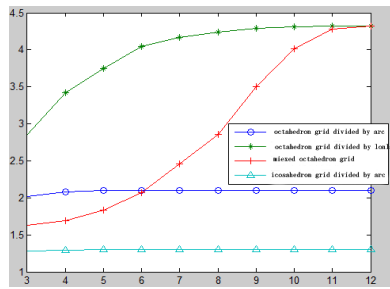


Figure 1. Max/Min area ratio

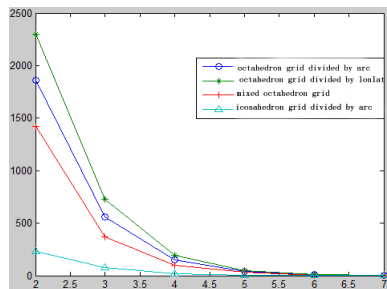


Figure 2. Mean square deviation of max/min area ratio

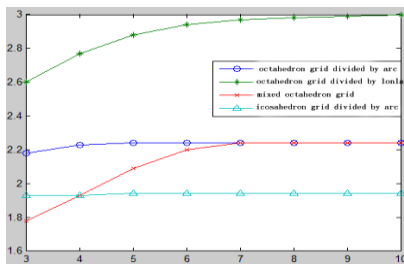


Figure 3. Major-minor axis ratio

must find out what the uniformity of the spherical grids is and how to measure it at first?

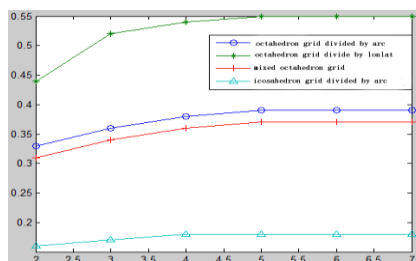


Figure 4. Mean square deviation of major-minor axis ratio

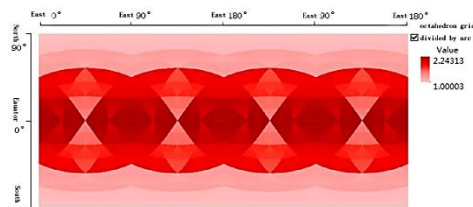


Figure 5. The planar distribution of octahedron grid divided by arc

surface of the globe starting with the icosahedron are better than others.

② The distortion characteristics of the angle and area of the spherical diamond discrete grids which are

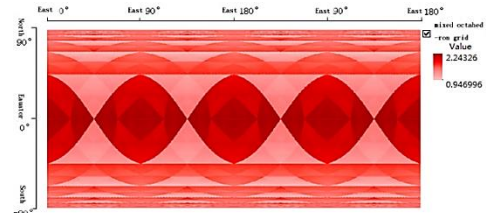


Figure 6. The planar distribution of octahedron grid divided by mixed method

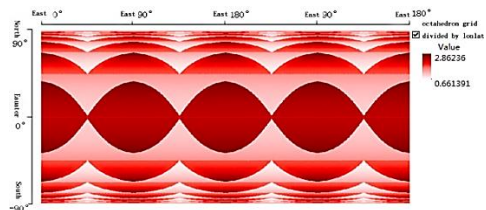


Figure 7. The planar distribution of octahedron grid divided by lonlat

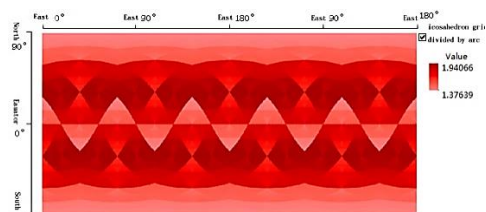


Figure 8. The planar distribution of icosahedron grid divided by mixed method

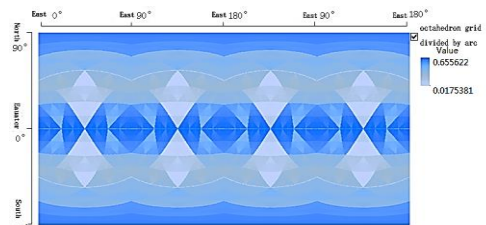


Figure 9. The planar distribution of mean square deviation of major-minor of octahedron grid divided by arc

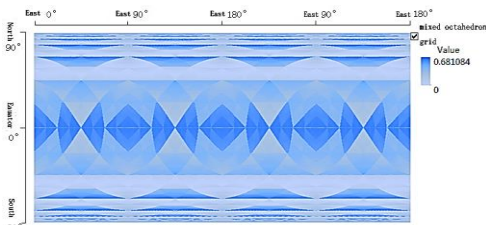


Figure 10. The planar distribution of mean square deviation of major-minor of octahedron grid divided by mixed method

obtained by recursively bisecting the longitude-latitude lines on the surface of the globe starting with the octahedron are worse than others.

③ The major-minor axis ratios and the distortion characteristics of angle of spherical diamond discrete grids

obtained by recursively bisecting arc and spherical diamond discrete grids obtained by recursively mixed bisecting method are the same when $n=7$, but the former is obvious much better than the latter, with regarding to distortion characteristics of area of the grid cell.

Researches above are mainly on the basis of the analysis of geometry characteristics of grid cells. In order to figure out

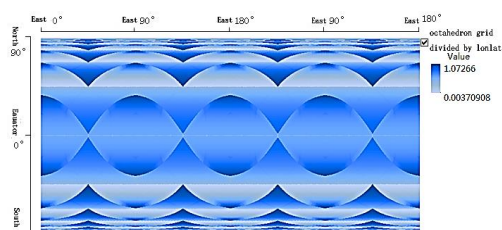


Figure 11. The planar distribution of mean square deviation of major-minor of octahedron grid divided by mixed method

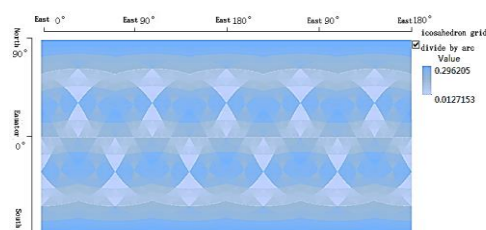


Figure 12. The planar distribution of mean square deviation of major-minor of icosahedron grid divided by arc

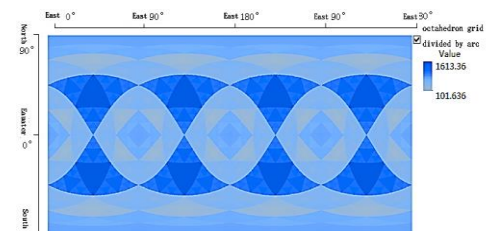


Figure 13. The planar distribution of mean square deviation of major-minor of octahedron grid divided by arc

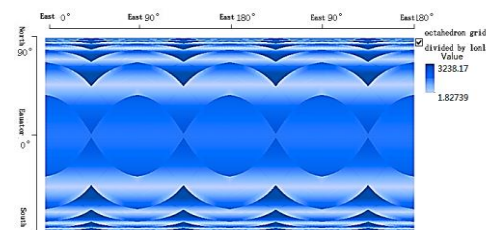


Figure 14. The planar distribution of mean square deviation of major-minor of octahedron grid divided by mixed method

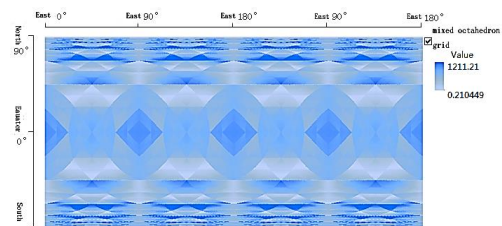


Figure 15. The planar distribution of mean square deviation of major-minor of octahedron grid divided by lonlat

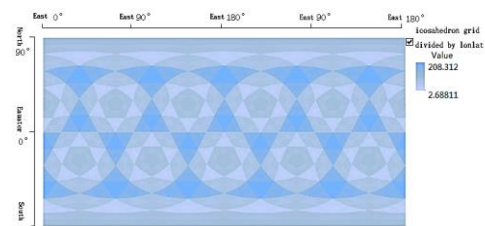


Figure 16. The planar distribution of mean square deviation of major-minor of icosahedron grid divided by arc

the geometry characteristics of the spherical grids comprehensively, we need to analyze the shape of the spherical diamond discrete grids from uniformity in addition to the analysis of the grid cell. Therefore, we

4. DISTORTION DISTRIBUTION OF SPHERICAL DIAMOND DISCRETE GRID

4.1 Electrical Potential Energy

According to the rule of nature, low energy system is easier to reach steady state. If we put N electric charges on the rigid spherical surface, the system energy, and the distribution of electric charges if uniform (at least, they are quasi-uniform). The total electrical potential energy equation (Jackson, 1999) is given by

$$U(r) = \sum_{i=1}^n \sum_{j=1, j \neq i}^n \frac{1}{4\pi\epsilon} \frac{q_i q_j}{r_{ij}} \quad (1)$$

Where r_{ij} is the spatial distance between q_i and q_j , q_i is electric quantity of electric charge in the centre of current position, q_j is electric quantity of electric charge in grid cells around q_i .

4.2 Experimental Analysis

From the above, we conclude that if we put the electric charges in the centre of the spherical diamond discrete grids, uniform grids have less electrical potential energy. Taking electrical potential energy as evaluation index of uniformity of the grid cells, we can draw following conclusions,

- ① The electrical potential energy and the mean square error of the electrical potential energy of the spherical diamond discrete grids which are obtained by recursively bisecting the great arcs on the surface of the

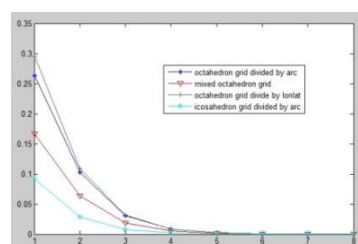


Figure 17. The electrical potential energy of the electrical potential energy

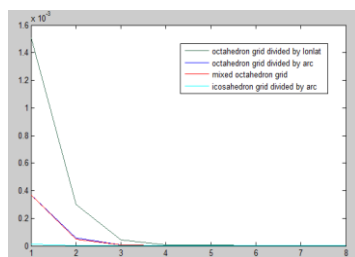


Figure 18. Mean square error of the electrical potential energy of the spherical diamond discrete grids

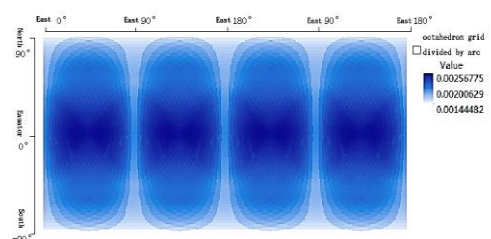


Figure 19. Distribution of electrical potential energy of spherical diamond discrete grids octahedron grid divided by arc

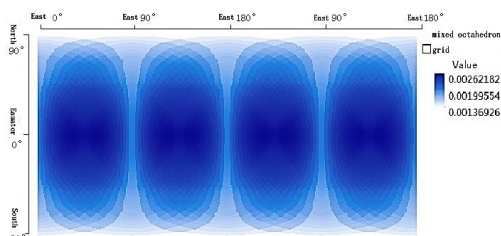


Figure 20. Distribution of electrical potential energy of spherical diamond discrete grids octahedron grid divided by mixed method

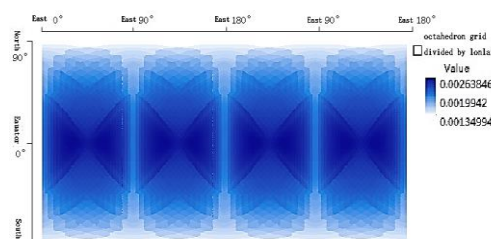


Figure 21. Distribution of electrical potential energy of spherical diamond discrete grids octahedron grid divided by lonlat

are obtained by recursively bisecting the great arcs on the surface of the globe starting with the icosahedron is the best.

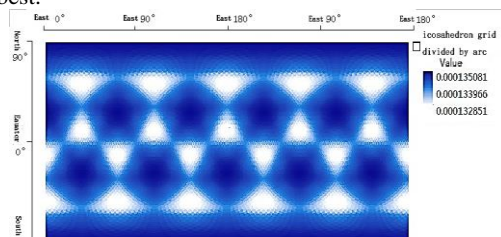


Figure 22. Distribution of electrical potential energy of spherical diamond discrete grids icosahedron grid divided by arc

globe starting with the icosahedron are lowest, they demonstrate that the uniformity of the spherical diamond discrete grids which

② The longitude-latitude lines on the surface of the globe starting with the octahedron are the highest, they demonstrate that the uniformity of spherical diamond

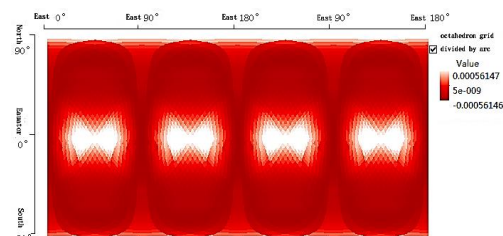


Figure 23. Distribution of electrical potential energy of spherical diamond discrete grids octahedron grid divided by arc

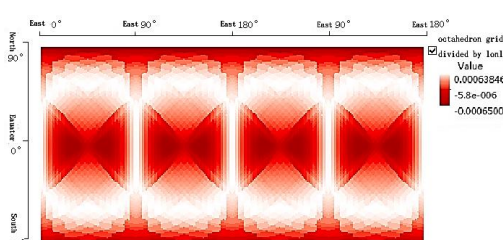


Figure 24. Distribution of electrical potential energy of spherical diamond discrete grids octahedron grid divided by lonlat

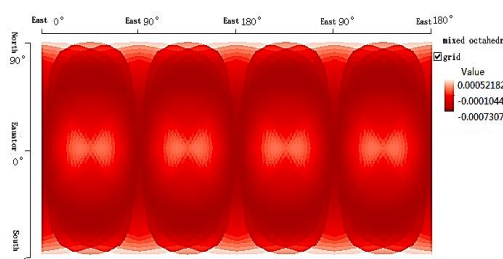


Figure 25. Distribution of electrical potential energy of spherical diamond discrete grids octahedron grid divided by mixed method

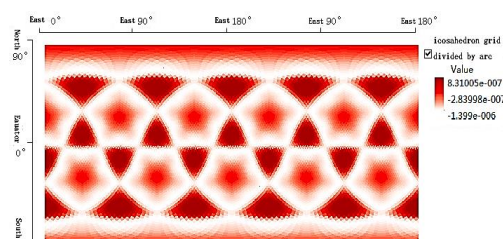


Figure 26. Distribution of electrical potential energy of spherical diamond discrete grids icosahedron grid divided by arc

discrete grids which are obtained by recursively bisecting the longitude-latitude lines on the surface of the globe starting with the octahedron are the highest, they demonstrate that the uniformity of spherical diamond discrete grids which are obtained by recursively bisecting the longitude-latitude lines on the surface of the globe starting with the octahedron is worst.

- ③ The electrical potential energy of spherical diamond discrete grids obtained by recursively bisecting arc and spherical diamond discrete grids obtained by recursively bisecting the longitude-latitude lines on the surface of the globe starting with the octahedron are nearly the same, but the mean square error of the electrical potential energy of former is obvious much better than that of the latter. The mean square error of the electrical potential energy of spherical diamond discrete grids obtained by recursively bisecting arc and spherical diamond discrete grids obtained by recursively mixed bisecting method on the surface of the globe starting with the octahedron are nearly the same, but the electrical potential energy of the former is lower than the latter, it demonstrates that the latter is steady during bisecting recursively.

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5. CONCLUSIONS

Bases on the above analysis, we can see that the spherical diamond discrete grids which are obtained by recursively bisecting the great arcs on the surface of the globe starting with the icosahedron are better than the others whether in distortion of the area of the grid cell and the distortion of the angle of the grid cell or from the global uniformity. Furthermore, the paper presents a measure method based on electrical potential energy of analyzing the global uniformity of the grids; the method can also be used to research the geometry characteristics of all the other kinds' grids.

6. ACKNOWLEDGEMENT

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