ON THE STANDARDIZATION OF VERTICAL ACCURACY FIGURES IN DEMS

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WG I/2, WG I/6, ICWG I/V, ICWG V/3, ICWG V/5

KEY WORDS: DEM/DTM, Accuracy, Error, Surface,

ABSTRACT:

Digital Elevation Models (DEMs) play a key role in hydrological risk prevention and mitigation: hydraulic numeric simulations, slope and aspect maps all heavily rely on DEMs. Hydraulic numeric simulations require the used DEM to have a defined accuracy, in order to obtain reliable results. Are the DEM accuracy figures clearly and uniquely defined? The paper focuses on some issues concerning DEM accuracy definition and assessment. Two DEM accuracy definitions can be found in literature: accuracy at the interpolated point and accuracy at the nodes. The former can be estimated by means of randomly distributed check points, while the latter by means of check points coincident with the nodes. The two considered accuracy figures are often treated as equivalent, but they aren't. Given the same DEM, assessing it through one or the other approach gives different results. Our paper performs an in-depth characterization of the two figures and proposes standardization coefficients.

1. INTRODUCTION

The paper focuses on some basic issues concerning DEM accuracy definition and assessment. The following terminology will be used: *DEM* is the mathematical reconstruction of a surface, regardless of what it represents: the bare terrain (DTM) or the terrain plus vegetation and buildings (DSM). *Mass points* are the (x, y, z) input data for DEM creation; they are usually measured by aerial lidar or photogrammetry, but can also be extracted from existing maps. A DEM is constituted by (x, y, z) points called *nodes* plus a structure. A DEM having a *TIN structure* is constituted by irregular nodes and a Delaunay triangulation; nodes usually coincide with the original mass-points, unless breaklines and constraints were imposed on the triangulation calculation. A DEM has a *GRID structure* when the nodes occupy the vertices of a regular grid, squared or rectangular. There are two levels of interpolation related to the calculation and use of a DEM. The *first interpolation* is used to estimate the nodes' height from that of mass points: this is a core process in GRIDs, while it is trivial for TINs. The *second interpolation* is used to estimate the height of any unknown point from the nodes.

Two DEM accuracy definitions can be found in literature: accuracy at the interpolated point and accuracy at the nodes, as detailed in Section 3. The former can be estimated by means of randomly distributed check points, while the latter by means of check points coincident with the nodes.

The two considered accuracy figures are often treated as equivalent, but they aren't. Our paper performs an in-depth characterization of the two figures and proposes **standardization coefficients**.

The paper is organized as follows: Section 2 focuses on related work; Sec. 3 properly defines the two accuracy figures; Sec. 4 carries out a detailed study of their stochastic properties; Sec. 5 discusses results and proposes standardization coefficients.

2. RELATED WORK

The *Guidelines for Digital Elevation Data* (NDEP, 2004), by the USA National Digital Elevation Program (NDEP) are a term of reference: they have inspired several other guidelines prepared by national agencies worldwide and are referenced by some scientific papers, as well. While the considered document defines and describes a number of interesting terms and topics, it doesn't really define what the DEM accuracy is. This definition is implicitly given when the accuracy assessment procedure is described, when the reader understands that the NDEP guidelines focus on the *interpolated height accuracy*. They recommend that, during the assessment procedure, TIN interpolation is performed for irregularly-distributed mass points and bilinear interpolation is applied for gridded DEMs: the guidelines disregard the different error-propagation properties of the two methodologies. Finally, they primarily focus on the *fundamental vertical accuracy* which is considered the most important DEM accuracy figure and can be estimated by choosing check points belonging to open areas, in flat terrain or uniform slope.

The ASPRS document (Flood, 2004) on *Vertical Accuracy Reporting for Lidar Data* is very similar to (NDEP, 2004): the same concepts of check points and fundamental accuracy are used; furthermore, the possibility of measuring check heights directly at the nodes is mentioned, but standardization coefficients are not mentioned.

(Kraus et al., 2004), (Kraus et al., 2006) and (Aguilar et al. 2010) are referenced only as authoritative examples of papers adopting the accuracy at the nodes definition.

(Frey & Paul, 2010) compare the ASTER and SRTM GDEMs with the Swiss National one named DHM25. Among the other analyses, they calculate vertical accuracy figures, using the accuracy at the nodes definition.

(Giribabu et al. 2013) generate a DEM by means of Cartosat-1 stereoscopic images, over the Himalayan area. They validate it by means of a number of CKPs, measured with dGPS, following the accuracy at the interpolated point concept.

Interestingly, there are papers mixing the two approaches. San & Suzen (2005) assess a number of DEMs extracted by them

This contribution has been peer-reviewed.
doi:10.5194/isprsarchives-XL-5-W3-73-2013

from an Aster stereo-couple over a Turkish region. They initially prepare a reference DEM, co-registered with the evaluated DEMs, by means of the contour lines of an existing 1:25000 map. They preliminary validate the reference DEM by comparison with 40 CKPs, extracted from a 1:25000 topographic map, using the accuracy at the interpolated point definition. The subsequent assessment of the Aster DEMs is performed instead by accuracy at the nodes, pixel by pixel.

(Shi et al. 2005) focus on variance propagation of bilinear and higher order interpolation methods. They only obtain a formula comparable with our (4). They don't highlight the relationship between the obtained results and assessment issues. (Zhu et al. 2005) deal with variance propagation of TIN interpolation, obtaining a result comparable to our formula (8). They don't obtain our functions (5) and (7) nor do they establish any connection between the variance propagation results and DTM assessment.

3. DEM VERTICAL ACCURACY AND ITS ASSESSMENT

DEM accuracy is a vast topic, involving several issues and error sources. Our discussion only concerns some items, disregarding others, such as planimetric and altimetric bias: we assume they have already been detected and fixed. Thus, the paper concerns vertical accuracy only.

But, **what is DEM vertical accuracy**? This concept has never been explicitly defined in papers and guidelines we have consulted, and is instead assumed. However, literature proposes two definitions of DEM vertical accuracy:

- *accuracy at the nodes*: the average vertical distance between the nodes and the terrain; when planimetric and altimetric biases are fixed, it only consists of the *RAndom Vertical Error at the Nodes* (RAVEN, σ_n), which is more precisely described below;
- *accuracy at the interpolated point*: the average distance between the generic interpolated point and the terrain.

The two definitions are clearly not equivalent, even if they are sometimes treated as such. Once biases are fixed, accuracy of the interpolated point is determined by three items:

a. the interpolation method;
b. *RAndom Vertical Error at the Point* (RAVEP, σ_P), which is the propagation of RAVEN;
c. the approximation error.

Approximation error describes the differences between the DEM discretized surface and the real one, independently of the existence of random errors: it depends on surface roughness and DEM spacing (nodes' average distance).

Let's now consider the parts of the terrain which are similar to a plane, in which the terrain is flat or has a uniform slope. The approximation error vanishes on those parts, provided that TIN or bilinear interpolation is used, which are the methods considered in the paper and most adopted in literature and guidelines. Under the above-listed conditions, accuracy at the interpolated point coincides with σ_P and is given by variance propagation of σ_n. The two figures are significantly different, as will be shown in the next Section, but this is apparently neglected by many documents in literature.

For the sake of clarity, a precise statistical definition of the considered problem is synthetically illustrated. The height of each node i is a random variable (RV) $Z_i[\mu_i, \sigma_n]$ having a certain true value μ_i, unknown, and a certain dispersion σ_n, the same for all nodes, corresponding to RAVEN; the actual node height z_i is an extraction from the RV Z_i; all the Z_i are assumed to be uncorrelated. Finally, there are no specific assumptions on the statistical distribution of the Z_i RVs.

A generic point (x_P, y_P) is now considered: its height can be obtained from the nodes' heights by interpolation. The interpolated height is another random variable $Z_P[\mu_P, \sigma_P]$ and is a function of the $Z_i[\mu_i, \sigma_n]$. The standard deviation σ_P (RAVEP) comes from variance propagation and can be formally evaluated, as the initial variance-covariance matrix is known

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_n^2 & 0 & 0 & 0 \\ 0 & \sigma_n^2 & 0 & 0 \\ 0 & 0 & \sigma_n^2 & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix}$$

as well as the interpolation function. The shown matrix is related to the GRID case, while in the TIN case it is 3×3, with the same structure.

The empirical accuracy assessment of a given DEM is now considered. This task is usually carried out by comparison with a number of check points. Most guidelines focus on *fundamental vertical accuracy* (NDEP, 2004) which is considered the most important DEM accuracy figure and can be estimated by choosing check points belonging to open areas, in flat terrain or uniform slope. From now the paper concerns fundamental vertical accuracy estimation.

Given n check points, whose *true* heights are z_i^{ckp}, the corresponding DEM heights are z_i^{DEM}. The differences can be formed

$$\delta_i = z_i^{DEM} - z_i^{ckp} \quad .$$

The empirical average can be evaluated, first of all

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \delta_i$$

and the empirical standard deviation can be further estimated

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (\delta_i - \hat{\mu})^2} \quad . \tag{1}$$

Two assessment schemas are now conceivable:

- check points are randomly chosen, according to the fundamental vertical accuracy prescription: in this case formula (1) is an estimation for the RAVEP, σ_P;
- check points coincide with nodes, therefore formula (1) is an estimation for RAVEN, σ_n.

Depending on the choice of check points, two different quality figures are estimated. The second considered possibility is only apparently difficult: using NRTK GPS, for instance, one can navigate to the planimetric position of one node and then measure the corresponding terrain height. Furthermore, when a second terrain model (DEM B) is available, more detailed and precise than the first one (DEM A), DEM B can be interpolated in order to obtain the *true* height of the DEM A nodes.

4. ERROR PROPAGATION IN DEM SECOND INTERPOLATION

Main results are summarized here, for error propagation in DEM second interpolation. Bilinear and TIN interpolation methods are considered, which are most used for DEM processing and analysis. The former is applicable to regular nodes only, and therefore solely to GRID-structured DEMs, while the second one can be applied to both TIN and GRID structures. Results shown are rigorously obtained in the most general situation, due to the adoption of a Computer Algebra System (CAS) such as the Matlab Symbolic Toolbox.

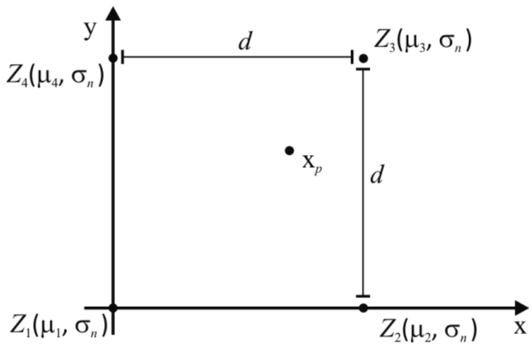

Figure 1. Bilinear interpolation.

4.1 Error propagation in bilinear interpolation

A square cell is considered in this section, having size d, whose lower left vertex is in the origin.

Figure 1 shows the four nodes considered, their heights and the generic (x_P, y_P) point. The interpolated height z_P has the form

$$z_P = a\,x_P + b\,y_P + c\,x_P y_P + e$$

Imposing the function to pass through the four nodes, the following analytical expression can be obtained

$$z_P(x_P, y_P; z_1, \cdots, z_4, d) = z_1 - \frac{1}{d}\left(x_P(z_1 - z_2) + y_P(z_1 - z_4)\right) + \\ + \frac{1}{d^2}\left(x_P y_P (z_1 - z_2 + z_3 - z_4)\right) \qquad (2)$$

All results shown in this section were obtained by symbolic calculations and carried out with the Matlab Symbolic toolbox: they don't contain any kind of approximations, as they are exact.

The behaviour of the obtained function is very well known from literature, so we are not discussing it; we only highlight that, at the centre of the cell, $z_P()$ is equal to the arithmetic mean of the four heights z_1, z_2, z_3, z_4. The analytical expression for σ_P can be obtained, as well

$$\sigma_P(x_P, y_P, d, \sigma_n) = \\ = \frac{\sigma_n}{d^2}\sqrt{\left(d^2 - 2d\,x_P + 2x_P^2\right)\left(d^2 - 2d\,y_P + 2y_P^2\right)} \qquad (3)$$

The function $\sigma_P()$ is not constant, meaning that the precision of the interpolated height depends on the position of the interpolated point within the cell. However, very interestingly, σ_P is independent from the nodes' height, that is, error propagation is independent from terrain's morphology. Figure 2 shows the plot of the function $\sigma_P()/\sigma_n$: the minimun value is $1/2$, at the centre of the cell (as the interpolated height is the simple mean of 4 independent measurements); the maximum value is 1, at the nodes.

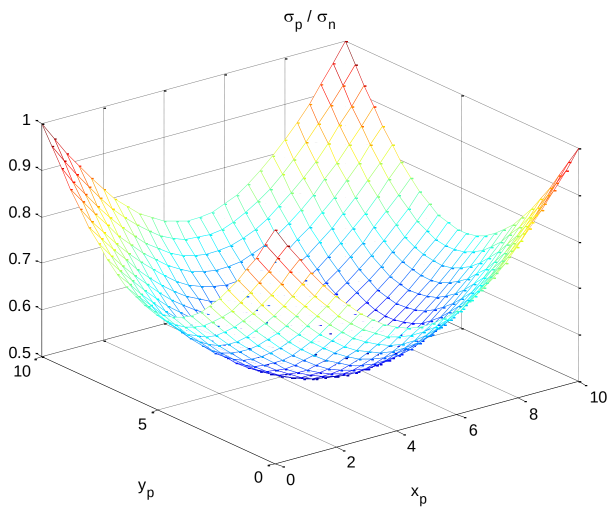

Figure 2 . Plot of the ratio σ_P/σ_n as a function of (x_P, y_P); cell size was set to 10 in order to produce the plot, but the function's shape is independent from d.

As it is useful to have a single figure quantifying the accuracy of the interpolated heights, function (3) can be averaged over the cell, in order to evaluate the average standard deviation. Actually we averaged the ratio

$$r^2 = \frac{\sigma_P^2()}{\sigma_n^2}$$

as calculations involving the variance are simpler, because of the absence of the square root. The average $\bar{r}^2$ coefficient can be calculated

$$\bar{r}^2 = \frac{1}{\sigma_n^2 d^2}\int_0^d \mathrm{d}x_P \int_0^d \mathrm{d}y_P\, \sigma_P^2(x_P, y_P, d, \sigma_n) = \\ = \frac{4}{9}$$

which, being a constant, is noticeably independent from d and σ_n. The average standard deviation of the interpolated height is therefore

$$\bar{\sigma}_P = \frac{2}{3}\sigma_n \cong 0.67\,\sigma_n \qquad . \qquad (4)$$

4.2 Error propagation in TIN interpolation

TIN interpolation is now studied. The considered triangle is shown in Figure 3 and has one vertex in the origin and another one on the *X* axis.

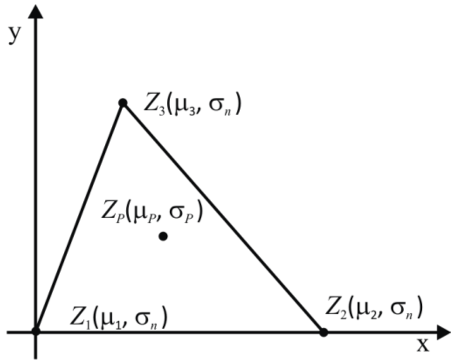

Figure 3. TIN interpolation.

The interpolated height is obtained by calculating the equations of the plane passing through the nodes (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and finding its height at the position (x_P, y_P):

$$z_P(x_P, y_P; z_1, x_2, z_2, x_3, y_3, z_3) = \frac{y_3 z_1 - y_P z_1 + y_P z_3}{y_3} + \\ + \frac{(x_3 y_P - x_P y_3)(z_1 - z_2)}{x_2 y_3} \tag{5}$$

When it is evaluated at the centre of the triangle, that is the point having coordinates

$$\frac{1}{3}(x_1 + x_2 + x_3, y_1 + y_2 + y_3) \tag{6}$$

the currently considered $z_P()$ function is equal to the arithmetic mean of the three heights z_1, z_2, z_3. The analytical expression for σ_P is

$$\sigma_P(x_P, y_P; z_1, x_2, x_3, y_3) = \\ = \sigma_n \sqrt{\left(\frac{x_P}{x_2} - \frac{x_3 y_P}{x_2 y_3}\right)^2 + \left(\frac{x_P}{x_2} - \frac{y_P(x_2 - x_3)}{x_2 y_3} - 1\right)^2 + \frac{y_P^2}{y_3^2}} \tag{7}$$

It depends on the position of P on the triangle, but is independent from the nodes' height, as it is in the bilinear case, and this means that error propagation is independent from terrain's morphology. It does depend, of course, on the triangle's shape. Figure 2 shows the plot of the function $\sigma_P()/\sigma_n$: the minimun value is $\sqrt{3}/3 \cong 0.58$, at the centre of the triangle and the maximum value is 1, at the nodes.

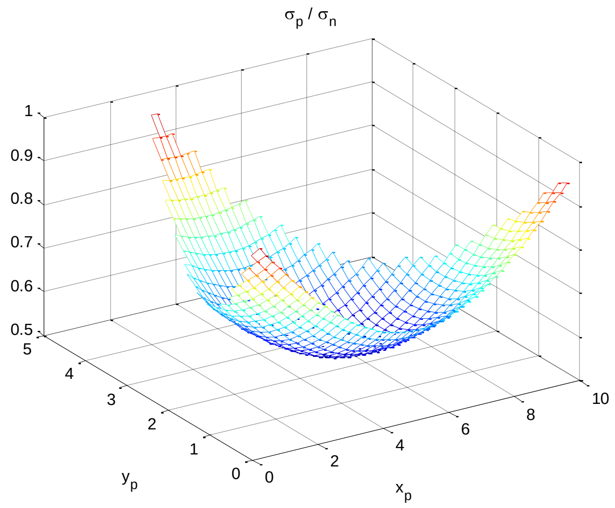

Figure 4. Plot of the ratio σ_P/σ_n as a function of (x_P, y_P), for TIN interpolation. In order to produce the plot, the triangle having vertices $(0,0)$, $(10,0)$ and $(3,5)$ was set, but the function's shape is independent from that.

The average $\bar{r}^2$ coefficient can be calculated

$$\bar{r}^2 = \frac{1}{\sigma_n^2} \frac{\iint_D \mathrm{d}x_P \mathrm{d}y_P \sigma_P^2(x_P, y_P; z_1, x_2, x_3, y_3)}{\iint_D \mathrm{d}x_P \mathrm{d}y_P} = \\ = \frac{1}{2}$$

where the integration domain D is the triangle. Noticeably, it is a constant, so it is independent on the shape of the triangle and on σ_n. The average standard deviation of the interpolated height is therefore

$$\bar{\sigma}_P = \frac{\sqrt{2}}{2}\sigma_n \cong 0.71\sigma_n \quad . \tag{8}$$

All results shown here refer to the triangle of Figure 3 but have a general value: thanks to the symbolic capabilities of Matlab, we also tested the only other significant configuration, in which the node number 3 is positioned on the right of number 2: results are the same. They can be summarized as follows: (i) the interpolated height is the simple mean of the three nodes' heights, when it is evaluated at the centre of the triangle; (ii) the standard deviation of the interpolated height is independent from the terrain's shape; (iii) the average standard deviation of the interpolated height is a constant, thus independent from the triangle shape.

5. DISCUSSION

A given DEM is considered, having a σ_n RAVEN. The model is interpolated at points which are in agreement with the fundamental vertical accuracy definition: open areas, flat terrain or uniform slope. When bilinear interpolation is used (requiring the GRID structure), the standard deviation of the interpolated points is, on average

$$\bar{\sigma}_P = \frac{2}{3}\sigma_n \cong 0.67\sigma_n \quad .$$

When TIN interpolation is adopted, the average standard deviation of the interpolated points is

doi:10.5194/isprsarchives-XL-5-W3-73-2013

$$\bar{\sigma}_P = \frac{\sqrt{2}}{2}\sigma_n \cong 0.71\sigma_n$$

There are three *accuracies* for DEM, and they are significantly different. We believe that RAVEN, accuracy at the nodes, must be considered the fundamental one.

When the empirical assessment is performed, three scenarios can be conceived:

1. check points are randomly distributed and bilinear interpolation is used; σ_n can be estimated by

   $$\hat{\sigma}_n = \frac{3}{2}\hat{\sigma} = 1.50\hat{\sigma} \quad \text{(see (1) for } \hat{\sigma}\text{)}$$

2. check points are randomly distributed and TIN interpolation is used; σ_n can be estimated by

   $$\hat{\sigma}_n = \sqrt{2}\,\hat{\sigma} = 1.41\hat{\sigma}$$

3. check points coincide with nodes (or a second DEM is used as check data); σ_n can be estimated directly

   $$\hat{\sigma}_n = \hat{\sigma}$$

6. CONCLUSIONS

The DEM vertical accuracy topic was tackled, assuming that planimetric and altimetric biases are removed and only parts of the terrain are considered, which are compatible with the fundamental vertical accuracy definition: open areas, flat terrain or uniform slope. GRID and TIN structures were considered, as well as bilinear and TIN interpolation.

Two main vertical accuracy definitions exist: accuracy at the nodes and accuracy at the interpolated point. The second one is a function of the position of the point within the cell or the triangle, and is in general significantly lower than the first one, because of the favourable error propagation and the average effect. Furthermore, accuracy at the interpolated point is slightly influenced by the interpolation methodology used.

When empirical accuracy assessment is performed by means of check points, depending on their position and the interpolation adopted, different accuracy figures can be obtained: they must be standardized with the shown coefficients, in order to have comparable results.

Further activities will concern numerical examples on simulated and real datasets.

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8. ACKNOWLEDGMENTS

The authors would like to thank the anonymous reviewers for their helpful comments.

The authors acknowledge the partial financial support of the University of Pavia, Italy (FAR 2012 grant).

Vittorio Casella performed the analysis described in Section 4 and wrote Sections 4 and 5. The remaining parts of the paper were jointly written by both the authors.

This contribution has been peer-reviewed.
doi:10.5194/isprsarchives-XL-5-W3-73-2013