HYPERBOLIC DISTORTION MODEL FOR RADIAL DISTORTION CORRECTION

[1]G.B. Ikokou, [2]J. Smit

[1]Tshwane University of Technology, Geomatics Department, Pretoria, South Africa,ikokougb@tut.ac.za

[2] University of Cape Town, Geomatics Division, South Africa,Julian.smit@uct.ac.za

KEY WORDS: Radial distortions, Hyperbolic model, Analytical distortion model, Camera calibration.

ABSTARCT

When information derived from the imagery is used for metric purposes and not as communication tools the impact of small imaging errors can be significant on the accuracy of derived information. As the number of photogrammetry applications grows and the technology advances, camera calibration becomes more complex. In fact, most lenses suffer from at least one kind of distortion profile such as barrel, pincushion or ‘moustache’ profiles. Commonly observed on wide angle lenses, barrel distortions curve straight lines inwards to give the image shape of barrel, however most lenses are prone to more than one distortion profile. Proposals for lens correction have been made using line-based approaches as well as division and rational models. The limitations of some of the techniques include the very limited number of distortion profiles they can handle due to their mathematical formulations. For example some can only handle distortion with positive coefficients and would exhibit instabilities when dealing with barrel distortions describe by negative distortion coefficients. In this paper we propose a new class of hyperbolic radial distortion model which handles several distortion profiles. The mathematical formulation of the new approach offers stability of the model since it can handle both positive and negative distortion coefficients -an improvement on some of the current techniques. The calibration results show that the approach produced the best distortion coefficients k_1 and k_2. The model can handle distortions from panorama imagery as well as ‘moustache ’profile produced by wide angle lenses.

1. INTRODUCTION

The development of consumer grade cameras and their integration in the Photogrammetry production environment make the need for calibration as important as before and more difficult to perform. The need for camera calibration, to establish interior orientation, has been a fundamental requirement since the foundation of photogrammetry. As the number of photogrammetry applications grows and the technology advances, camera calibration becomes more complex. In fact most lenses suffer from at least one kind of distortion profile such as barrel, pincushion or ‘moustache’. Commonly observed on wide angle lenses, barrel distortions curve straight lines inwards to give the image a shape of barrel. On the other hand pincushion distortions make straight lines curved outwards from the centre of the image. However, the most complex of lens distortions is the ‘moustache’ distortion profile which is basically the combination of barrel and pincushion profiles. Straight lines appear curved inwards towards the centre of the frame then curve outwards at the extreme corners. For this reason, ‘moustache’ distortions are sometimes referred to as having a complex profile. Functional models implemented in Software packages have been proposed in the literature to deal with lens distortions. However, in order for a software to deal with specific distortions related to a specific lens, the distortion profile must be built in the software database, making camera calibration a very challenging task when dealing with various or complex profiles. Following these, it is evident that the efficiency of camera calibration is related to the mathematical formulation of the distortion model. This paper presents a ‘hyperbolic’ radial distortion model to deal with large distortion magnitudes. The mathematical formulation of the model offers an advantage of handling moustache as well as individual barrel and pincushion profiles. The presence of both the sine and cosine hyperbolic functions in the model offers the property of handling asymmetric radial distortions.

2. RELATED STUDIES

Tommaseli et al., (2014) tested five camera calibration techniques for wide angle lenses. The models include the perspective, stereographic, equidistant, equi-solid angle and the orthogonal models with additional parameters. The perspective model relates the image coordinates to the camera coordinates through a rigid body transformation that does not account for the translation between origins of coordinate systems and is given as follows:

$$\begin{bmatrix} x_i - x_0 \\ y_i - y_0 \\ 1 \end{bmatrix} = -f \begin{bmatrix} X - X_{cp} \\ Y - Y_{cp} \\ Z - Z_{cp} \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ 0 \end{bmatrix} \quad (1)$$

With $r_{11}, r_{12,} r_{13} ... r_{33}$ the elements of the rotation matrix and X, Y, Z the coordinates of the 3D point in the world

https://doi.org/10.5194/isprs-archives-XLII-2-W13-1667-2019

coordinate system and X_{cp}, Y_{cp}, Z_{cp} the coordinates of the origin of the world coordinate system. The x_0, y_0 coordinates represent the coordinates of the image centre while $\Delta x, \Delta y$ the sum of lens distortions applied to the x and y image coordinates given by the following:

$$\Delta x = x\left(k_1 r^2 + k_2 r^4 + k_3 r^6\right) + p_1\left(r^2 + 2x^2\right) + 2p_2 xy + A_x + B_x$$
$$\Delta y = y\left(k_1 r^2 + k_2 r^4 + k_3 r^6\right) + p_2\left(r^2 + 2y^2\right) + 2p_1 xy + A_y + B_y \quad (2)$$

Where k_1, k_2, k_3 the radial distortion coefficients and p_1, p_2 the coefficients of decentring distortions and A, B the affinity parameters. The perspective model differs from the other models in its mathematical formulation but also in terms of the sign of the focal length measure as illustrated in the equation (1). From the above equation the 3D and the 2D image planes are viewed from the lens centred coordinate system which results in the focal length and the $z-axis$ pointing to opposite directions. All tested camera models were implemented in a software package to solve the model parameters using the self-calibration bundle adjustment technique relying on at least seven absolute geometric constraints. The results demonstrated that the perspective model is not suitable for fish eye lenses since it produced the highest standard deviation in terms of coefficient residuals and focal length residuals as well as the highest residuals of affinity parameters.

The camera model proposed by **Tsai (1987)** only models radial distortion. The accuracy of this method was reported sufficient for most of photogrammetry applications **(Horn, 2000).** Due to its simplicity the proposed model is reported not suitable to correct complex distortions such as those produced by wide angle lenses. **Weng (1992)** proposed a two steps calibration approach in which the transformation from the 3D space coordinate to the 2D image coordinates is firstly modelled by a perspective projection composed of a rotation and a translation as follows:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T \quad (3)$$

Where R is a 3×3 rotation matrix defining the camera orientation and T is a translation vector defining the camera position. The transformation between the image coordinates and the camera coordinates is given by the following:

$$\begin{bmatrix} u \\ v \end{bmatrix} = f \begin{bmatrix} \frac{x_c}{z_c} \\ \frac{y_c}{z_c} \end{bmatrix} \quad (4)$$

As geometrical distortions affect the positions of image points in the image plane, the relations in (4) do not hold true and should be replaced by the relations in (5) as follow:

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} u + \delta_u(u,v) \\ v + \delta_v(u.v) \end{bmatrix} \quad (5)$$

Where u , v are the image undistorted coordinates and u', v' their respective distorted coordinates. The terms δ_u and δ_v describe the radial distortions applied to the u and v coordinates given by the expressions as follows:

$$\begin{bmatrix} \delta_u \\ \delta_v \end{bmatrix} = \begin{bmatrix} k_1 u\left(u^2 + v^2\right) \\ k_1 v\left(u^2 + v^2\right) \end{bmatrix} \quad (6)$$

Wang et al., (2009) proposed a line-based technique which employs rational models derived from the traditional Brown's radial distortion model and described in **Fitzgibbon (2001).**The model relates the distorted and undistorted points on an image by the following functions:

$$x_u = \frac{x_d}{1 + \lambda r^2} \text{ and } y_u = \frac{y_d}{1 + \lambda r_d^{\,2}} \quad (7)$$

Where λ is the first coefficient of radial distortion. These produce the equation of the undistorted line as follows:

$$\frac{y_d}{1 + \lambda r_d^{\,2}} = a\frac{x_d}{1 + \lambda r_d^{\,2}} + b \quad (8)$$

After reformulation the relation in (8) gives:

$$y_d = ax_d + b + b\lambda\left(x_d^{\,2} + y_d^{\,2}\right) \quad (9)$$

Which after simplification produces the following:

$$x_d^{\,2} + y_d^{\,2} + \frac{a}{b\lambda}x_d - \frac{1}{b\lambda}y_d + \frac{1}{\lambda} = 0 \quad (10)$$

The equation (10) is the equation of a circle showing the graphics of a distorted straight line **(Wang et al., 2009).** By extracting three straight lines from the image and by determining parameters that satisfy their equations then substituting those parameters into (10) the technique can recover the radial distortion coefficients. Although the technique was reported efficient to correct severe distortions but it only deals with symmetric type of radial distortions.

Ahmed and Farag (2005) proposed a technique based on the slope of a line. The approach uses the pinhole model to project the 3D point from the world coordinate system onto the image plane as follows:

$$\begin{bmatrix} x_u \\ y_u \\ 1 \end{bmatrix} = \begin{bmatrix} x_w \\ y_w \\ z_o \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

With x_w, y_w, z_w the coordinates of a point in the 3D scene and f the camera constant. The measured point's coordinates are related to their corresponding undistorted estimates through the equations (12) and (13) as follows:

$$x_u = x_d + (x_d - x_0)\left[k_1 r_d^2 + k_2 r_d^4 + ...\right] + \left[p_1 \times ...\right] \tag{12}$$
$$y_u = y_d + (y_d - y_0)\left[k_1 r_d^2 + k_2 r_d^4 + ...\right] + \left[p_2 \times ...\right] \tag{13}$$

With $r_d^2 = (x_d - x_0)^2 + (y_d - y_0)^2$ and $k_1, k_2 ...$ the coefficients of radial distortion while $p_1 ...$ the coefficients of decentring distortion. A line straightness constraint is proposed to associate undistorted points with their corresponding distorted coordinates such that any points $p_i(x_i, y_i)$ on a line l must satisfy its equation expressed as:

$$ax_u + by_u + c = 0 \tag{14}$$

Since the expressions in (12) and (13) relate the undistorted point to the distorted point's coordinates the equation in 14 can be written as a function of measurable image coordinates as follows(**Ahmed and Farag,2005**):

$$f(x_d, y_d) = a\left[x_d + (x_d - x_0)(k_1 r^2 ...)\right] + ... \tag{15}$$

By calculating the elemental change of f (δf) from equation (15) one obtains the equation (16):

$$\delta f = a\left[\frac{\partial x_u}{\partial x_d}\delta x_d + \frac{\partial x_u}{\partial y_d}\delta y_d\right] + b\left[\frac{\partial y_u}{\partial x_d}\delta x_d + \frac{\partial y_u}{\partial y_d}\delta y_d\right] \tag{16}$$

This equation represents the tangent to the distorted curve and its slopes can be estimated by the equation [17] as follows:

$$s(x_d, y_d) = \frac{\dfrac{\partial y_u}{\partial x_d} + \dfrac{\partial y_u}{\partial y_d}\dfrac{\delta y_d}{\delta x_d}}{\dfrac{\partial x_u}{\partial x_d} + \dfrac{\partial x_u}{\partial y_d}\dfrac{\delta y_d}{\delta x_d}} \tag{17}$$

Given a chain of edge points on the distorted line $\left(x_d{}^i, y_d{}^i, i = 1,2,....N\right)$, the error E between points' positions on the distorted line can be estimated through the squared difference between slopes of the lines expressed as follows:

$$E = S\left(x_d^i, y_d^i\right)^2 - S\left(x_d^{i-1}, y_d^{i-1}\right)^2 \tag{18}$$

In the case where several lines are considered on the distorted image the error in (18) would be estimated as the sum of errors as follows:

$$E = \sum_{i=2}^{n}\left(S\left(x_d^i, y_d^i\right) - S\left(x_d^{i-1}, y_d^{i-1}\right)\right)^2 \tag{19}$$

Under the correct values of the distortion parameters, the error estimated in (19) should be zero. This error can be minimized using non-linear optimization algorithm starting with initial estimated values of distortion parameters. Although the proposed distortion error was reported efficient, the distortion model itself assumes a symmetry of radial distortion and produces strong correlation between distortion parameters and the polynomial model only corrects distortions at image corners (Jacobsen, 2003).

3. METHODOLOGY

3.1. Calibration points

The calibration points were extracted from images of a calibration filed located in the Geomatics department at University of Cape Town. The estimation of the near error free image coordinates were done using the collinearity equation (Tagoe et. al., 2014) given as follows:

With x_u, y_u the near error free image coordinates while X_S, Y_S, Z_S are the coordinates of a point on the 3D scene viewed from the camera frame coordinate system. The elements of the rigid body transformation matrix are given by $r_{11}, r_{12} ... r_{33}$ which describe the rotation matrix while T_x, T_y, T_z the translation vectors.

3.2. Distortion model

Projection errors created by camera lenses can project straight lines as curves on the image sensor. The geometric interpretation of circular functions is well known but that of their hyperbolic counterparts does not appear equally well known in Photogrammetry literature. The importance of 'hyperbolic' trigonometry for the study of radial distortion does not appear widespread known while the functions hold potential when describing curves. The original model proposed in this study is given by the expression in (20) as follows:

$$r_u = r_d\left[\frac{e^{k_1 r^2 + k_2 r^4} + e^{-k_1 r^2 - k_2 r^4}}{2} + \frac{e^{k_1 r^2 + k_2 r^4} - e^{-k_1 r^2 - k_2 r^4}}{2}\right] \tag{20}$$

With r_u, r_d the undistorted and distorted radial distances and k_1, k_2 the coefficients of radial distortion and r the distance between the distorted and its undistorted location.

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After simplification the model in (20) can be expressed as follows:

$$r_u = r_d \left[\frac{2e^{k_1 r^2 + k_2 r^4}}{2} \right] \quad (21)$$

Estimating the ratio between the undistorted and distorted radial distances produces the distortion error as follows:

$$\frac{r_u}{r_d} = e^{k_1 r^2 + k_2 r^4} \quad (22)$$

Applying the natural logarithm on the ratio produces the linear equation (23) as follows:

$$k_1 r^2 + k_2 r^4 = \ln\left(\frac{r_u}{r_d}\right) \quad (23)$$

Provided at least two image points' coordinates and their near error free estimates the distortion coefficients k_1 and k_2 can be solved analytically.

3.3. Results validation

To validate our approach we implemented the method of Ahmed and Farag (2005). The distortion error was computed as squared slopes difference between distorted and undistorted lines. The rational model in Wang et al., (2009) was modified by adding a second distortion coefficient for a more objective accuracy comparison in addition to the two parameters polynomial model. Table1 presents the distortion coefficients' estimates computed from the set of points extracted from the panorama imagery. An analysis of the table shows that the imagery exhibits 'moustache' distortion profile described by apposite signs of both the distortion coefficients k_1 **and** k_2 **(Tang et al., 2017).** In terms of accuracy of the first distortion coefficient k_1 the line based method was the less efficient for the lenses used in this study while the best results were obtained by the proposed hyperbolic and rational models as illustrated in figure1.

Line-based	Rational	Polynomial	Hyperbolic
1.3×10^{-2} (k_1)	5.9×10^{-5} (k_1)	6.2×10^{-5} (k_1)	2.9×10^{-5} (k_1)
-4.7×10^{-8} (k_2)	-2.7×10^{-8} (k_2)	-2.8×10^{-8} (k_2)	-7.7×10^{-6} (k_2)

Table 1: Distortion coefficients' estimates

When it comes to the second radial distortion coefficient k_2 the polynomial model offered better results than the rational model while the 'hyperbolic' model minimized the best the projection error on the imagery as illustrated in figure2.

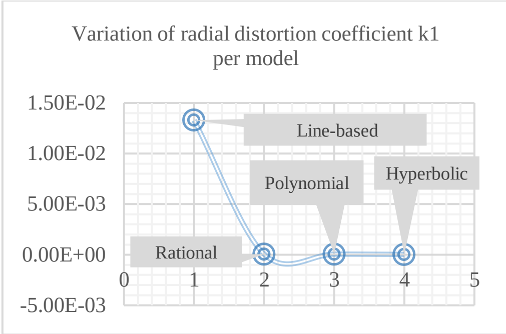

Figure 1: Variation of distortion coefficient per model

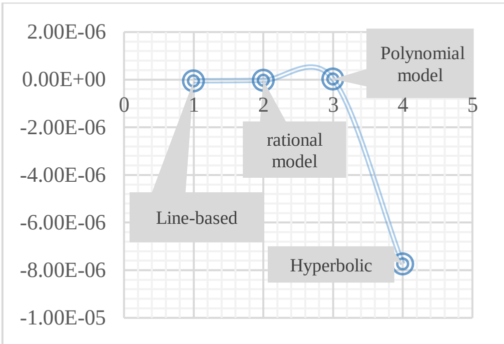

Figure 2: Variation of distortion coefficient k_2 per model.

4. CONCLUSION.

This study tested the performance of a 'hyperbolic' radial distortion model for asymmetric radial distortion. The model showed good performance when dealing with distortion produced by wide angle lenses. The polynomial model offered the second best estimate of the second distortion coefficient, outperforming the irrational model. This may originate from the mathematical formulation of the model which has no distortion parameter on the numerator's expression. The limited performance of the line based approach could originate from the formulation of the distortion error as other approaches used ratios of undistorted and distorted distances which minimize best the error than the squared difference of slopes. When dealing with severe distortions produced large discrepancies between slopes could create numerical instabilities on the estimated coefficients. The proposed model performance could be improved with more image points as the points with large projection errors importantly influenced the averaged estimates of the coefficients with the line based technique. The proposed model's performance can be improved with more image points as points with large errors importantly influenced the estimates of distortion coefficients. This study could be extended by adding more distortion coefficients and tested against more models as the performance comparison was only done against three of the currently proposed techniques in the literature.

5. REFERENCES.

Ahmed, M. and Farag, A., 2005. Nonmetric calibration of camera lens distortion: differential methods and robust estimation. *IEEE Transactions on image processing*, *14*(8), pp.1215-1230.

Fitzgibbon, A.W., 2001. Simultaneous linear estimation of multiple view geometry and lens distortion. In *Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition. CVPR 2001* (Vol. 1, pp. I-I). IEEE.

Horn, B.K., 2000. Tsai's camera calibration method revisited. *Online: http://people. csail. mit. edu/bkph/articles/Tsai_Revisited. pdf*.

Tang, Z., von Gioi, R.G., Monasse, P. and Morel, J.M., 2017. A precision analysis of camera distortion models. *IEEE Transactions on Image Processing*, 26(6), pp.2694-2704.

Tommaselli, A.M.G., Marcato Jr, J., Moraes, M.V.A., Silva, S.L.A. and Artero, A.O., 2014. Calibration of panoramic cameras with coded targets and a 3D calibration field. *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences*, *40*(3), p.137.

Tsai, R., 1987. A versatile camera calibration technique for high-accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses. *IEEE Journal on Robotics and Automation*, *3*(4), pp.323-344.

Wang, A., Qiu, T. and Shao, L., 2009. A simple method of radial distortion correction with centre of distortion estimation. *Journal of Mathematical Imaging and Vision*, *35*(3), pp.165-172.

Weng, J., Cohen, P. and Herniou, M., 1992. Camera calibration with distortion models and accuracy evaluation. *IEEE Transactions on Pattern Analysis & Machine Intelligence*, (10), pp.965-980.

Zhou, F., Cui, Y., Gao, H. and Wang, Y., 2013. Line-based camera calibration with lens distortion correction from a single image. *Optics and lasers in Engineering*, *51*(12), pp.1332-1343.

https://doi.org/10.5194/isprs-archives-XLII-2-W13-1667-2019