AN INTEGRATED NETWORK-CONSTRAINED SPATIAL ANALYSIS FOR CAR ACCIDENTS: A CASE STUDY OF TEHRAN CITY, IRAN

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ABSTRACT:

Research on determination of spatial patterns in urban car accidents plays an important role in improving urban traffic safety. While traditional methods of spatial clustering of car accidents mostly rely on the two dimensional assumption, many spatial events defy this assumption. For instance, car accidents are constrained by the road network and rely on the one dimensional assumption of street network. The aim of this study is to detect and statistically prioritize the car accident-prone segments of an urban road network by a network-based point pattern analysis. The first step involves estimating the density of car accidents in the one dimensional space of the road network using the network kernel density estimation (NKDE) method with equal-split continuous and discontinuous kernel functions. In the second step, due to the lack of statistical prioritization of the accident-prone segments with NKDE method, the output of the NKDE method is integrated with network-constrained Getis-Ord Gi* statistics to measure and compare the accident-prone segments based on the statistical parameter of Z-Score. The integration of these two methods can improve identification of accident-prone segments which is effective in the enhancing of urban safety and sustainability. These methods were tested using the data of damage car accidents in Tehran District 3 during 2013-2017. We also performed the Network K-Function to display the significant clustering of damage car accident points in the network space at different scales. The results have demonstrated that the damage car accidents are significantly clustered.

1. INTRODUCTION

Today, increasing car accidents has a negative impact on the people's life and social development. Therefore, a huge amount of efforts should be made to strengthen traffic safety standards in order to develop sustainable transportation (Flak, 1997). Spatial pattern analysis can provide an effective solution to identify global or local spatial distribution patterns of urban car accidents (Prasannakumar et al., 2011). Therefore, spatial pattern analysis has been used in urban road network based on geospatial information systems (GIS) in order to identify accident-prone segments of car accidents. GIS has been used as a science and technology for modeling and analysis of car accidents (Deepthi Jayan and Ganeshkumar, 2010). Spatial pattern analyses can be classified into local and global methods (O'Sullivan and unwin, 2014). The first category examines the severity of accidents and determines the absolute location of discrete events, such as planar kernel density estimation (PKDE) method (Anderson, 2009; Erdogan et al., 2008). The second category examines the spatial interaction of discrete events for spatial patterns, such as nearest neighbor statistics, K-Function methods in both planar and network modes and Getis-Ord Gi* statistics (Okabe and Yamada, 2001; Chaikaew et al., 2009; Flahaut et al., 2003). The PKDE method is a non-parameter method which has been widely used for analyzing the discrete events. Although no single technique has been achieved as the best method for detection of car accident clusters, recent research suggests that the PKDE method offers a better output due to its simplicity and ease of execution (Carlos et al., 2010; Pulugurtha et al., 2007). The PKDE method when the accident locations are sufficient for the analysis, performs appropriately to identify accident-prone segments of the road network (Yu et al., 2014). Although the PKDE method has shown acceptable properties using density values, its two dimensional assumption is not logical for discrete events that are distributed in the one dimensional space (Yamada and Thill, 2004). To overcome this limitation, Okabe et al. (1995) proposed NKDE method for estimating kernel density in network space, which can overcome the weaknesses of the PKDE method and provides more logical results. Although PKDE and NKDE are useful methods for analyzing car accidents, they have some limitations. One of their main limitations is the lack of prioritization of the accident-prone segments in the road network (Bil et al., 2013). Therefore, it is necessary to decide which clusters are statistically significant. The aim of this research is to detect and prioritize accident-prone segments of the urban road network. Therefore, the NKDE method is used to discover the high-density road segments in the network space. Then, the network-constrained Getis-Ord Gi* statistics is used to prioritize the accident-prone segments of the road network based on the statistical parameter with inputs obtained by the NKDE method. Network K-Function method is also used to identify clustering of point events in different distance intervals of the network space.

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2. DATA AND METHODS

2.1 Data

In this paper, we investigated the urban area of Tehran District 3 that covers an area of 29.27 km^2 . The road network is 725888.12 meters with 7200 road segments. The study area is located between $35^{\circ}44'30''$ N - $35^{\circ}48'00''$ N Latitude and $51^{\circ}22'30''$ E - $51^{\circ}28'30''$ E Longitudes. The data used in this research is related to location of damage car accidents in Tehran District 3 during 2013-2017 as shown in Figure 1. There have been about 9662 damage car accidents during 2013-2017.



Figure 1. damage Car accidents damage location in Tehran District 3 during 2013-2017

2.2 Methods

The flowchart of the proposed method is shown in Figure 2.



Figure 2. Flowchart of the proposed method for car accidents clustering

2.2.1 Network Kernel Density Estimation (NKDE)

Network Kernel Density Estimation method (NKDE) has been used to detect cluster pattern of point events in the one dimensional space. According to Figure 3, the main difference between the NKDE and PKDE methods is that in the NKDE method in addition to considering network space alongside point events, the shortest network distance is used instead of Euclidean distance in an one dimensional space (Xie and Yan, 2008).



Figure 3. The difference between the NKDE and PKDE methods (a) PKDE (b) NKDE (Xie and Yan, 2008).

The estimate of the density of the point events in the network space is calculated according to Equations 1 and 2 (Xie and Yan, 2008):

$$L(s) = \sum_{q=1}^{n} \frac{1}{r} k(\frac{d_s(q,p)}{r})$$
(1)

$$k\left(\frac{d_s(q,p)}{r}\right) = K(1 - \frac{d_s(q,p)^2}{r^2})$$
(2)

Where r = search radius

q = kernel center

p = observed points

k = Base kernel function (Quartic kernel)

K = 0.75

 $d_s(q,p)$ = the network shortest distance between the kernel center (q) and the observed accident locations.

In this method, the choice of the two parameters r and k is of great importance. As the search radius increases, the density level becomes smoother. According to previous research for urban car accidents, the search radius of 100 meters was selected (Steenberghen et al., 2010; Erdogan, 2015). The key feature of the NKDE method is that the road network is divided into basic linear units (BLU) with equal network length called Lixel (corresponding to pixels in the planar space), which is associated with the network topology. Using Lixel not only selects a set of locations with regular intervals along the network to estimate the density, but also significantly improves the efficiency of the computation (Xie and Yan, 2008). Okabe et al. (2009) Suggested that the length of Lixel is $(\frac{\text{search radius}}{10})$ as a rule of thumb. The 10 NKDE method is defined based on the following steps (Xie and Yan, 2008):

- 1. a certain threshold is considered to check the topology and connectivity of the road network.
- 2. Each segment of the road network is divided into basic linear units (Lixel).
- 3. The kernel function and the search radius are used to determine the estimation of density car accidents in each Lixels.
- 4. Calculate the shortest-path network distance from the center of each source Lixel to centers of all its neighboring Lixels within the search radius.

- 5. At the center of each source Lixel and all its neighboring Lixels, calculate a density value based on a selected kernel function, the network distance, and the number of events on the source Lixel and assign the total density to the source Lixel.
- 6. Output the density value of each Lixels.

The topology changes in the road network nodes reduces the accuracy of the density in the nodes. Hence, to calculate the density around the nodes, two types of kernel functions, including Equal-split continuous kernel function and Equal-split discontinuous kernel function have been used (Okabe et al., 2009). For Equal-split discontinuous kernel function when the center of the kernel function (q) does not reach the node (is located near the intersection), the kernel function is defined as Equation 3 (Okabe et al., 2009):

$$K_{q}(p) = \begin{cases} \frac{k(\frac{d_{s}(q,p)}{r})}{(n_{i1}-1)(n_{i2}-1)\dots(n_{ik-1})} & a \le d_{s}(q,p) < b \\ 0 & d_{s}(q,p) \ge h \end{cases}$$
(3)

Where $K_q(p) =$ kernel function

h = search radius n = degree of nodes on the network $a = ds (q, v_{ik-1}), b = ds (q, v_{ik})$

Otherwise, when the center of kernel (q) reaches a node (q = vi1), the value of the kernel function is defined as Equation 4, whose parameters are the same as those of Equation 3 (Okabe et al., 2009):

$$K_{q}(p) = \begin{cases} \frac{2k(\frac{d_{s}(q,p)}{r})}{(n_{i_{1}})(n_{i_{2}}-1)\dots(n_{i_{k-1}})} & a \le d_{s}(q,p) < b \\ 0 & d_{s}(q,p) \ge h \end{cases}$$
(4)

Equal-split discontinuous kernel function is shown in Figure 4 (Okabe et al., 2009):



Figure 4. Equal-split discontinuous kernel function (Okabe et al., 2009)

For the second type assuming that a node in the distance h is from the center of kernel (q) and $v_1 = v_{i1}$, $n_1 = n_{i1}$, kernel function is calculated according to Equation 5 (Okabe et al., 2009):

$$K_{q}(p) = \begin{cases} k(d_{s}(q,p)) & \text{for } -h \leq d_{s}(q,p) < 2c - h \\ k(d_{s}(q,p)) - \frac{n_{1} - 2}{n_{1}} k(2d_{s}(q,v_{s1}) - d_{s}(1,p)) \\ & \text{for } 2c - h \leq d_{s}(q,p) < c \\ \frac{2}{n_{1}} k(d_{s}(q,p)) & \text{for } c \leq d_{s}(q,p) < h \end{cases}$$
(5)

where $c = d_s(q, v_l)$

Other parameters are the same as those of Equation 3 and 4. Equal-split continuous kernel function is shown in Figure 5 (Okabe et al., 2009):



Figure 5. Equal-split continuous kernel function (Okabe et al., 2009):

The major limitation of NKDE method is that there is no statistical significance parameter to prioritize the accident-prone segments of the road network (Xie and Yan, 2008). Therefore, Getis-Ord Gi* statistics is used.

2.2.2 Network-Constrained Getis-Ord Gi* Statistics

Getis-Ord Gi* statistics was introduced to evaluate the local spatial pattern. Getis-Ord Gi* statistics is used to identify accident-prone clusters based on statistical significance. The output of this method for each feature has two terms including Z-Score and P-Value, which are used to calculate the statistical significance of spatial autocorrelation between any feature and its neighbors (Getis and Ord, 1992; Ord and Getis, 1995). The Z-Score value is a standard deviation representing the clustering of features with high or low values together, and P-Value represents the randomness or non-randomness of spatial patterns. According to Figure 6, The values of P-Value and Z-Score can have different states as follow (Mitchell, 2005):

- high Z-Score and small P-Value indicates clustering of high values.
- small Z-Score and small P-Value indicates clustering of low values.



Figure 6. Different states of P-value and Z-Score (Mitchell, 2005)

A very small P-Value represents the notion that the observed spatial pattern is not the result of random processes and the null hypothesis is rejected. The values of P-value and Z-Score are related to standard normal distribution (Griffith, 2008). The statistical index of Getis-Ord Gi* was introduced by Getis and Ord in 1992 to study the local pattern of spatial data and was developed in 1995 (Getis and Ord, 1992; Ord and Getis, 1995). The statistical index of Getis-Ord Gi* and Z-Score are presented in Equation 6 to 10 (Ord and Getis, 1995):

$$G_{i}^{*} = \frac{\sum_{j=1}^{n} W_{ij}(d) x_{j} - \frac{\sum_{j=1}^{n} x_{j}}{n} \sum_{j=1}^{n} W_{ij}(d)}{S_{n} \sqrt{\frac{[n \sum_{j=1}^{n} W_{ij}(d)^{2} - (\sum_{j=1}^{n} W_{ij}(d))^{2}]}}$$
(6)

$$z_{G_{l}} = \frac{G_{l}^{*} - E[G_{l}^{*}]}{\sqrt{V[G_{l}^{*}]}}$$
(7)

$$S = \sqrt{\frac{\sum_{j=1}^{n} x_j^2}{n} - (\frac{\sum_{j=1}^{n} x_j}{n})^2}$$
(8)

$$E[G_i^*] = -\frac{\sum_{i=1}^n \sum_{j=1}^n W_{ij}(d)}{n(n-1)}$$
(9)

$$V[G_i^*] = E[G_i^{*2}] - E[G_i^*]^2$$
(10)

Where x_j = attribute value for feature j

n =total number of features

 W_{ij} = a symmetric one/zero spatial weight matrix from a threshold d for the distance between features i and j.

Network-constrained Getis-Ord Gi* Statistics is based on Getis-Ord Gi* Statistics; however, the definition of weight matrix is different. Therefore, in Network-constrained Getis-Ord Gi* Statistics, weight matrix defines the neighboring relationships between the two network segments. Two types of weight matrices exist including the node-based and the distance-based matrix. In the node-based method, two segments are neighbors if they share a node. The distance-based matrix determines the neighboring relationships based on whether the distance between the midpoints of segments is less than a distance threshold or not (Yamada and thill,2007; Borruso, 2008).

2.2.3 Network K-Function method

The planar K-Function method identifies clusters in space beyond the road network. The network K-Function method was introduced to overcome this problem (Okabe and Yamada, 2001). The Network K-Function method is an extension of planar K-Function technique that uses the shortest path between two points to compute the network distance of the points and measure the spatial patterns of point events at different scales of the network space (Rui et al. 2015). The Network K-Function denoted by K(t), is defined as Equation 11 (Okabe and Yamada, 2001):

$$K(t) = \frac{1}{\frac{n-1}{2}} \frac{\sum_{i=1}^{n} n(t|p_i)}{n}$$
(11)

Where n = number of point events

S = Length of subnetwork (Lixel)

 $n(t/p_i)$ = The number of point events in the shortest network distance between the point event *t* and p_i .

In this method, Monte Carlo simulation is used to evaluate the spatial pattern of point events in the network space. In order to verify the clustering of point events, observations of the Network K-Function and mean expected values are compared. If the observations of the Network K-Function are higher than the expected values, then the set of point events is in the cluster distribution and the complete spatial randomness (CSR) hypothesis will be rejected. Otherwise, if the observations of the Network K-Function are lower than the expected values, then the set of point events is in the dispersion distribution (Ni et al., 2016)

3. RESULTS

3.1 NKDE method

In order to implement the NKDE method, the threshold to check the connection segments of the road network is set to 0.001 meter. Also, the length of Lixels is set to 10 meters and the search radius is set to 100 meters (Xie and Yan, 2008; Steenberghen et al., 2010; Erdogan, 2015). Figure 7, illustrates the vulnerability map of damage car accidents with the two kernel functions based on the density parameter of the damage car accidents. The density-based maps are classified into five output classes according to standard deviation classification method which is shown qualitatively from very low density to very high density.



Figure 7. The output of the NKDE method (a) continuous kernel function (b) discontinuous kernel function

According to Table 1, for damage car accidents the NKDE method with continuous kernel function identifies more density

segments than those of the NKDE method with discontinuous kernel function in the road network.

Kernel type	Total length of the density	Total length of the road	Percentage of density segments			
	segments (Meter)	network (Meter)	in road network			
Continuous kernel function	250279.83	725888.12	34.47			
Discontinuous kernel function	220583.76	725888.12	30.38			

Table 1. Comparison of the length density segments resulting from the NKDE method with the two kernel functions

3.2 Network-Constrained Getis-Ord Gi* Statistics

To implement the Network-Constrained Getis-Ord Gi* statistics, density values resulting from NKDE method is used as an attribute for computing the Network-Constrained Getis-Ord Gi* statistics. In order to define the weight matrix, the distance-based method is used. The threshold for computing the spatial weight was chosen 300 meters based on highest Z-Score. Also to detect accident-prone segments at 99% confidence level, the simulation count for Monte Carlo simulation is set to 999 times. According to Table 2, the 10 accident-prone segments resulting from the integration of NKDE method and Network-Constrained Getis-Ord Gi* statistics with two different kernel functions are prioritized according to the Z-Score statistical parameter.

Continuous kernel function				Discontinuous kernel function							
Order	Segment	Density	Z-Score	Number	District	order	Segment	Density	Z-Score	Number	District
	number	value	(Mean)	of			number	value	(Mean)	of	
		(Mean)		Lixels				(Mean)		Lixels	
1	5943	684.99	37.60	27	niayesh	1	5943	633.98	34.60	27	niayesh
2	2146	528.25	26.61	16	niayesh	2	2146	466.42	23.28	16	niayesh
3	6512	405.77	20.79	36	niayesh	3	6512	350.35	17.55	36	niayesh
4	1166	325.22	14.62	26	niayesh	4	5530	288.98	14.62	12	niayesh
5	5530	245.89	12.54	12	niayesh	5	1166	303.82	13.92	26	niayesh
6	2833	262.18	10.50	26	niayesh	6	1167	207.23	12.14	25	niayesh
7	4646	151.41	9.58	13	niayesh	7	1786	193.87	9.80	12	niayesh
8	4272	131.85	8.28	22	niayesh	8	2833	227.64	9.01	26	niayesh
9	2223	124.57	8.13	13	Kurdistan	9	4646	135.35	8.83	10	niayesh
10	1167	127.36	7.78	25	niayesh	10	3526	157.54	7.58	21	hemmat

 Table 2. Comparison of the accident-prone segments resulting from the integration of NKDE method and Network-Constrained

 Getis-Ord Gi* statistics with two different kernel functions

The classification of the Z-Score and P-Value statistical parameters for identify of the accident-prone segments is based on Table 3 (Mitchell, 2005). Figures 8 and 9, represent the vulnerability map of the accident-prone segments resulting from the integration of NKDE method and Network-Constrained Getis-Ord Gi* statistics based on the Z-Score parameter at 99%

confidence Level. The Z-Score parameter by the NKDE method and Network-Constrained Getis-Ord Gi* with Equal-split continuous kernel function were in the interval [-2.32, 51.56] and the Z-Score calculated by the NKDE method and Network-Constrained Getis-Ord Gi* with Equal-split discontinuous kernel function were in the interval [-2.28, 48.66].

Z-Score	P-Value	G _i *	Confidence levels (%)
< -1.65 or > +1.65	< 0.1	+/-1.0	90
< -1.96 or > +1.96	< 0.05	+/-2.0	95
< -2.58 or > +2.58	< 0.01	+/-3.0	99

Table 3. P-Values and Z-Scores for detection of accident-prone segments at different confidence levels (Mitchell, 2005)



Figure 8. The vulnerability map of the accident-prone segments resulting from the integration of NKDE method (continuous kernel function) and Network-Constrained Getis-Ord Gi* statistics based on Z-Score at 99% confidence level (a) 2D (b) 3D



Figure 9. The vulnerability map of the accident-prone segments resulting from the integration of NKDE method (discontinuous kernel function) and Network-Constrained Getis-Ord Gi* statistics based on Z-Score at 99% confidence level (a) 2D (b) 3D

After producing the vulnerability maps of the damage accidents, the number of dangerous Lixels (Z-Score >2.58) detected in the accident-prone segments of the road network at 99% confidence level has been compared. Therefore, according to Table 4, it can be verified that the integration of the NKDE method (continuous kernel function) and Network-Constrained Getis-Ord Gi* statistics at 99% confidence level identifies more dangerous Lixels than those of the integration of NKDE method (discontinuous kernel function) and Network-Constrained GetisOrd Gi* statistics in the density segments of the road network. Also, the length of the accident-prone segments resulting from integration of the NKDE method (continuous kernel function) and Network-Constrained Getis-Ord Gi* statistics is greater than the those of the integration of the NKDE method (discontinuous kernel function) and Network-Constrained Getis-Ord Gi* statistics. The Z-Score parameter frequency chart for dangerous Lixels with the two kernel functions is shown in Figure 10.

input	Confidence level (%)	Number of dangerous Lixels in the accident- prone segments	Z-Score mean for accident- prone segments	Total length of the accident- prone segments (Meter)	Total length of density segments (Meter)	Percentage of accident-prone segments in density segments
NKDE	99	952	7.17	7501.78	250279.83	2.99
(continuous)						
NKDE	99	780	7.27	6552.47	220583.76	2.97
(discontinuous)						

Table 4. Comparison of the results from the integration of NKDE method and Network-Constrained Getis-Ord Gi* statistics with the two kernel functions



Figure 10. Comparison of Z-Score parameter frequency chart for dangerous Lixels of the accident-prone segments (Z-Score>2.58) with the two kernel functions (a) continuous kernel function (b) discontinuous kernel function

3.3 Network K-Function method

In order to implement the network K-function method, the number of iterations for Monte Carlo simulation to identify clustering of point events in different distance intervals at 99% confidence level is set to 999 times.



Figure 11. Network K-Function Analysis for the damage accidents

In Figure 11, the horizontal axis shows the distance range (Meter) and vertical axis shows the cumulative number of car accidents. Considering that the observations of the Network K-Function (blue cure) are higher than the mean expected values (red cure), with a 99% confidence interval, the damage car accidents are significantly clustered.

4. CONCLUSION

In this research, NKDE method was used to identify the accidentprone segments of the road network. The key feature of the NKDE method is that the road network is divided into basic linear units called Lixel, which is associated with the network topology. Due to the change of network topology in the nodes, the density of the network nodes is calculated based on the two Equal-split continuous and Equal-split discontinuous kernel functions. As verified in the NKDE method, along with the density of car accidents, the density of the road network has been also taken into account. Considering that in the NKDE method there is no statistical parameter to prioritize the accident-prone segments of the road network, the Network-Constrained Getis-Ord Gi* statistics was used to prioritize the accident-prone segments at 99% confidence level based on the Z-Score statistical parameter. It was found that integration of the NKDE (continuous kernel function) and Network-Constrained Getis-Ord Gi* statistics at 99% confidence level identifies more accident-prone segments than those of the integration of NKDE (discontinuous kernel function) and Network-Constrained Getis-Ord Gi* statistics in the road network. Finally, the Network K-Function method was used to investigate the clustering of the damage car accidents at different interval scales. The damage car accidents in the urban road network space were shown is in the cluster distribution. In addition, the random distribution hypothesis of the data is rejected. For future research, it is suggested that other spatial autocorrelation methods be used to prioritize accidentprone segments of the road network and compared with the Network-Constrained Getis-Ord Gi* statistics in terms of the detection of accident-prone segments at different confidence levels.

REFERENCES

Flak, M. A., 1997. Reducing accident impacts under congestion In traffic Congestion and traffic Safety in the 21st Century: Challenges, Innovations, and Opportunities Urban Transportation, Division; ASCE.

Prasannakumar, V., Vijith, H., Charutha, R., Geetha, N., 2011. Spatio-temporal clustering of road accidents: GIS based analysis and assessment. Procedia-Social Behavioral Science , 21, 317– 325.

Deepthi Jayan, K., Ganeshkumar, B., 2010. Identification of accident hot spots: a GIS based implementation for Kannur District, Kerala. International Journal of Geomatics and Geoscience 1 (1), 51-59.

O'Sullivan, D., Unwin, D., 2014. Geographic Information Analysis, 2nd ed.; JohnWiley & Sons: Hoboken, NJ, USA.

Anderson, T.K., 2009. Kernel density estimation and K-means clustering to profile road accident hotspots. Accident Analysis & Prevention. 41(3), 359–364.

Erdogan, S., Yilmaz, I., Baybura, T., Gullu, M., 2008. Geographical information systems aided traffic accident analysis system case study: City of Afyonkarahisar. Accident Analysis & Prevention, 40, 174–181.

Okabe, A., Yamada, I., 2001. The K-Function method on a network and its computational implementation. Geographical Analysis, 33(3), 271–290.

Chaikaew, N., Tripathi, N.K., Souris, M., 2009. Exploring spatial patterns and hotspots of diarrhea in Chiang Mai, Thailand. International of Journal Health Geographic's. 8(1), 36.

Flahaut, B., Mouchart, M., Martin, E.S., Thomas, I., 2003. The local spatial autocorrelation and the kernel method for identifying black zones: A comparative approach. Accident Analysis & Prevention, 35, 991–1004

Carlos, H.A., Shi, X., Sargent, J., Tanski, S., Berke, E.M., 2010. Density estimation and adaptive bandwidths: A primer for public health practitioners. International of Journal Health Geographic's, 9(1), 39.

Pulugurtha, S.S., Krishnakumar, V.K., Nambisan, S.S., 2007. New methods to identify and rank high pedestrian car zones: An illustration. Accident Analysis & Prevention. 39, 800–811.

Yu, H., Liu, P., Chen, J., Wang, H., 2014. Comparative analysis of the spatial analysis methods for hotspot identification. Accident Analysis & Prevention, 66, 80–88.

Yamada, I., Thill, J.C., 2004. Comparison of planar and network K-Functions in car accident analysis. Journal of Transport Geography, 12(2), 149–158.

Okabe, A., Yomono, H., Kitamura, M., 1995. Statistical analysis of the distribution of points on a network. Geographical Analysis. 27(2), 152–175.

Bil, M., Andrasik, R., Janoska, Z., 2013. Identification of hazardous road locations of car accidents by means of kernel

density estimation and cluster significance evaluation. Accident Analysis & Prevention. 55, 265–273.

Xie, Z., Yan, J., 2008. Kernel density estimation of car accidents in a network space. Computers, Environment and Urban System. 35 (5), 396–406.

Okabe, A., Satoh, T. and Sugihara, K., 2009. A Kernel density estimation method for Networks, its computational method and a GIS-Based tool. International Journal of geographical information science, 23(1), 7-32.

Steenberghen, T., Aerts, K., Thomas, I., 2010. Spatial clustering of events on a network. Journal of Transport Geography, 18(3), 411-418.

Erdogan, M., Ilci, V., Soysal, O., Korkmaz, A., 2015, A model suggestion for the determination of the traffic accident hotspot on the Turkish highway road network: a pilot study Boletim de Ciencias Geodesicas.21(1), 169-188.

Getis, A., Ord. J.K., 1992. The Analysis of Spatial Association by Use of Distance Statistics in Geographical Analysis 24(3).

Ord, J.K., Getis, A., 1995. Local spatial autocorrelation statistics: Distributional issues and an application. Geographical Analysis, 27(4), 286–306.

Mitchell, A., 2005. The ESRI Guide to GIS Analysis: Volume 2: Spatial Measurements & Statistics (Vol. 2). Redlands, CA: ESRI press.

Griffith, D. A., 2008. Spatial-filtering-based contributions to a critique of geographically weighted regression (GWR), Environment and planning, 40(11), 2751-2769.

Yamada, I., Thill, J.C., 2007. Local indicators of network constrained clusters in spatial point patterns. Geographical Analysis, 39, 268–292.

Borruso, G., 2008. Network density estimation: A GIS approach for analysing point patterns in a network space. Transactions in GIS, 12, 377–402.

Rui, Y., Yang, Z., Qian, T., Khalid, S., Xia, N., Wang, J., 2015. Network-constrained and category-based point pattern analysis for Suguo retail stores in Nanjing, China. International Journal of Geographical Information Science, 30(2), 186–199.

Ni, J., Qian, T., Xi, C., Rui, Y., & Wang, J., 2016. Spatial distribution characteristics of healthcare facilities in Nanjing: Network point pattern analysis and correlation analysis. International journal of environmental research and public health, 13(8), 833.