Positioning Improvement for Spaceborne Laser Footprint Based on Precisely Terrain Data

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Abstract
Spaceborne laser altimetry represents a novel active remote sensing technology applicable to earth observation, which together with imaging spectroscopy and synthetic aperture radar as a core technology for data acquisition in the earth observation systems. However, the accuracy of horizontal positioning for laser footprints from spaceborne laser altimeters declines due to various factors such as the changes in the orbital environment and the deterioration of performance. Moreover, the limited frequency of in-orbit calibration of the spaceborne laser altimeters and the non-disclosure of calibration parameters mean that users are heavily reliant on positioning accuracy of the altimetry data provided. To address this issue, a new algorithm is proposed in this study for enhancing the accuracy of horizontal positioning for laser footprints in the absence of satellite altimeter pointing and ranging parameters. In this algorithm, high-resolution DSM is taken as the reference terrain data to take advantage of the higher precision in elevation over horizontal positioning of the laser footprints. By adjusting the horizontal position of the laser footprint within a small area, the algorithm achieves the optimal alignment of laser elevation data with the reference terrain. Then, the resulting shift in the horizontal position of the laser footprints is referenced to correct their horizontal positioning during that period. Based on the high-accuracy DSM data collected from the Xinjiang autonomous region in China and the data collected by the GF-7 satellite, simulation experiments are performed in this study to analyze and validate the proposed algorithm. According to the experimental results, the horizontal accuracy of the laser footprints improves significantly from 12.56 m to 3.11 m after optimization by the proposed method. With the elimination of 9.45 m horizontal error, accuracy is improved by 75.23%. This method is demonstrated as effective in further optimizing the horizontal position of laser altimetry data products in the absence of altimeter parameters and original data, which promotes the application of spaceborne laser data.

1. Introduction

The precise positioning of satellite laser footprints on the ground plays an essential role in the application of satellite laser altimetry data, laying a foundation for various scientific studies and applications that rely on this data. However, there are a range of factors that can cause the coordinates of the laser footprints to shift during the operation of laser altimetry satellites in orbit, such as thermal effects, the aging of instruments, and platform vibrations, as calculated using the onboard geometric positioning model of the laser footprint. These shifts severely compromise the quality of laser altimetry data. Research has shown that given the terrain with a slope of 1°, a 30-second pointing deviation of the laser altimeter at an altitude of 600km can lead to a horizontal error of up to 87 meters and a vertical error of about 1.5 meters. Considering the errors in laser ranging, these errors could potentially increase to more than 2 meters. Therefore, the geometric calibration of the laser altimeter is crucial to reducing the errors in pointing and ranging and to enhancing the accuracy of satellite laser altimetry products.
To resolve this issue, plenty of studies have been conducted on the precise positioning of satellite-borne laser footprints. For instance, there is a method proposed for on-orbit geometric calibration of satellite-borne laser altimeters, which requires the use of ground-based laser detectors (Magruder, Schutz et al. 2001, Magruder, Schutz and Silverberg 2003, TANG Xinming 2021). On this basis, Magruder et al. proposed a method in which corner cube retroreflectors (CCR) are applied to on-orbit calibration (Magruder, Schutz et al. 2001, Magruder, Silverberg et al. 2005, Magruder, Webb et al. 2007). This method involves placing CCRs at the top of different poles at varied heights. With different heights used to simulate terrain undulations, time differences are created in the echo. Based on the time information and CCR echo intensity information within the digital waveform, the centroid location of the laser footprint is calculated through CCR signal analysis. Furthermore, Schutz et al. proposed an aircraft infrared camera imaging calibration method(Schutz 2001, Magruder, Ricklefs et al. 2010). According to this method, the aerial infrared imagery obtained from aircraft is used to process data and ascertain the spatial relationship between laser spots and control points. Thus, the on-orbit calibration of satellite-borne laser altimeters is facilitated. These methods all eliminate the need to determine the centroid of the laser footprint using ground-based instruments. Then, the pointing and range correction parameters of satellite-borne laser altimeters are calibrated. Additionally, the methods in another category take full advantage of natural terrain features to perform on-orbit geometric calibration of satellite-borne laser altimeters. One of them involves satellite attitude maneuver calibration based on various flat terrains such as sea or flat land to achieve angular calibration(Rowlands, Pavlis et al. 1999, Luthcke, Rowlands et al. 2000, Luthcke, Carabajal and Rowlands 2002, Luthcke, Rowlands et al. 2005). This method relies on satellite attitude maneuvers to measure satellite laser altimeter range by means of ocean scans (OS) or attitude maneuvers across entire orbits ("Round"-The-World Scans, RTWS). It also involves Bayesian least squares differential correction to minimize the number of times for residual range measurement of the altimeter. Thus, the calibration of pointing and ranging is facilitated for the laser altimeter. Another method requires waveform matching calibration, which is intended specifically for linear-mode laser altimetry satellites (Martin,

Thomas et al. 2005). This technique matches recorded waveforms with predefined templates to fine-tune the calibration parameters of the altimeter. A highlight of this method lies in the calibration of pointing and range correction for satellite-borne laser altimeters through waveform analysis. Additionally, due to the considerable distance between the satellite and earth, even the slight deviations in pointing can cause significant horizontal errors. On non-flat terrains, the larger the horizontal error, the lower the precision of laser altitude measurement. Given different pointing angles at which ground elevation measurement is performed, the elevation values of the same area vary. Hence, only the elevation calculated with the correct pointing angle is comparable to the actual terrain data. By following this principle, a terrain-matching-based method has been proposed in some studies for on-orbit geometric calibration of satellite laser altimeters (Filin 2001, Li, Tang et al. 2017, Tang, Xie et al. 2019). In this method, the measured terrain is matched with known terrain data to correct any discrepancies in the readings by the altimeter. At present, this approach has attracted widespread attention from scholars due to its ease of operation and high success rate. Liu et al. demonstrated that high-precision terrain reference data can be used to improve the accuracy of terrain matching (Liu, Xie et al. 2022). Zhao et al. developed a new terrain matching algorithm by aligning laser ranging profiles with terrain contours (Zhao, Li et al. 2022). Additionally, Gao et al. applied a Random Forest (RF) model to analyze the factors affecting ICESat-2 satellite data, with insights gained into the influencing variables of ICESat-2 data (Gao, Xing et al. 2023). Xu et al. conducted research on the factors affecting the methods of terrain matching calibration, with a technique developed for correcting the errors in range correction (Xu, Mo et al. 2024, Xu, Xie et al. 2024). These achievements reflect the ongoing efforts invested to enhance the precision and reliability of satellite laser altimetry through in-depth terrain analysis and sophisticated data processing techniques. In the process of developing the aforementioned calibration methods, the focus is placed mainly on calibrating the pointing and ranging correction parameters of the satellite-borne laser altimeters, which is largely due to the fact that the current methods of locating laser footprints require the use of a geometric positioning model based on the satellite-to-earth geometric relationship (TANG Xinming 2021). In this model, the inherent geometrical configuration between the satellite and the earth's surface is referenced to accurately locate each laser footprint, which ensures that the readings by the laser altimeter are precise and reliable. In this model, the high-precision positioning of laser footprints is achievable if there are precise pointing and ranging correction parameters known for the laser altimeter. However, it is common that the pointing and ranging correction parameters are not publicly available for satellite-borne laser altimeters. As a result, it is particularly challenging for the users of laser altimetry data to accurately determine the coordinate positions of the laser data through the aforementioned methods, especially the horizontal positions. Due to a lack of access to crucial calibration data, the applicability and accuracy of the laser altimetry data are constrained in various settings, such as topographic mapping, forest canopy studies, and others in relation to environmental monitoring. During satellite operations, there are various factors that can significantly reduce the accuracy of horizontal positioning for laser data relative to altimetry. For example, during the validation of measurement time and geographic positioning time for the Geoscience Laser Altimeter System (GLAS) onboard LiDAR altimetry system, Magruder found out that the positional error for the laser products 2a and 3a was 10.6 m ±4.5 m and 7.5 m ±6.6 m, respectively (Magruder, Silverberg et al. 2005). In comparison, the altimetry precision of GLAS reached up to 0.15 m. For the Global Ecosystem Dynamics Investigation (GEDI), which can achieve an altimetry precision of 0.6 m (Fayad, Baghdadi et al. 2020), the precision of its laser footprint positioning reaches merely 10 m or so (Zhang, Chen et al. 2022) . According to the research of Xie et al., the altimetry data collected from the GF-7 (Gaofen-7) satellite has an elevation accuracy of 0.1 m in flat areas (TANG Xinming 2021, Xie, Liu et al. 2023), but its horizontal accuracy is roughly in the range of 5-8 m.

To address this issue, a novel algorithm is proposed in this paper for the improved horizontal accuracy of laser footprints without requiring the parameters from satellite-borne laser altimeters. In fact, the change in elevation within a small horizontal shift of a laser footprint is far less significant than its horizontal change. In light of this, the proposed method takes advantage of the higher elevation accuracy of laser altimetry data compared to its horizontal accuracy. The algorithm is initiated by creating a grid of equidistant horizontal coordinates centered around the initial horizontal positioning coordinates of the laser. Then, it traverses these grid points without altering the elevation values, interpolates the elevation of laser footprints at different coordinates using a high-precision Digital Surface Model (DSM), and calculates the difference between the laser elevation and the interpolated DSM elevation. By locating the point with the minimum elevation difference, the optimal offset for the laser footprint is determined. Thus, the horizontal position of the laser footprint is optimized. In this study, GF-7 altimetry data is taken as the research subject to validate this method by optimizing the GF-7 laser data subjected to noise.

2. Research Data

2.1 GF-7 Laser Data

In November 2019, China succeeded in launching the GaoFen-7 (GF-7) satellite, China's first sub-meter level stereo mapping satellite, with the aim of 1:10000 scale stereo mapping and large-scale updates on geographic information data (Tang, Xie et al. 2020). This GF-7 satellite adopts a dual-beam laser altimetry system, with an angle of 0.7° formed between the nadir points of the two beams. Laser is emitted to create a footprint that is approximately 30 meters in diameter on the surface of earth. The consecutive laser footprints created on the surface of earth are about 2.4 km apart from each other. Meanwhile, the two beams are further separated in the lateral direction, with a spacing of about 12.25 km. In the dual-beam laser emission system, each altimeter can be used to emit the laser with a wavelength of 1064 nm at a frequency of 3 Hz (Xie, Huang et al. 2020). The data used in this study for GF-7 Beam2 laser data relate pertains to its transit over the Kuche region in Xinjiang, China, on June 7, 2021. The details are shown in Table 1.

Table 1 GF-7 Beam 2 Laser Data

Date	Beam	Orbit	Time Code
2021.6.7	Beam 2	8851	234538605.671431
			234538606.004764
			234538606.671431
			234538607.004764
			234538607.338097
			234538607.671431
			234538608.004764
			234538608.671431
			234538609.004764

2.2 Reference Terrain Data

In 2021, an aerial remote sensing survey was conducted to collect high-precision topographic data from a specific area. For the survey, a manned fixed-wing aircraft complete with the Leica Terrain Mapper airborne LiDAR system was deployed. The airplane maintained an altitude of 5200 meters and a maximum speed of 309 km/h along the designated route. Throughout the flight, the LiDAR system operated at a pulse emission frequency of 436,000 Hz and a scanning frequency of 61.3 Hz, thus ensuring the high temporal and spatial resolution of the laser point cloud data. After data collection, filtering and post-processing, the DSM with a resolution of 1 m and an elevation accuracy of 0.17 m was obtained. Figure 1 shows the location, elevation, and shape of the DSM data collected from the KuChe area located in Xinjiang. Since the route followed by the airplane is consistent the trajectory of the GF-7 satellite, the DSM data is presented as parallelogram.

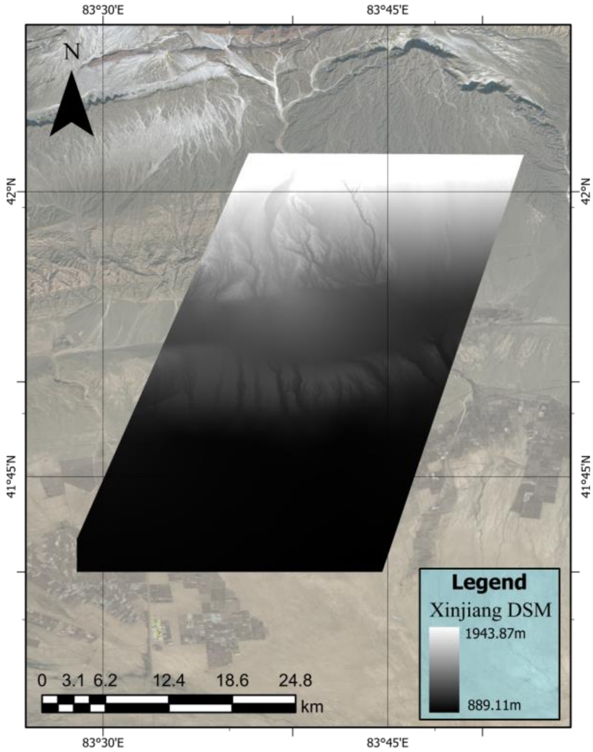

Figure 1 DSM data collected from the Kuche region of Xinjiang.

3. Methodology

3.1 Modelling of the changes in laser footprint level and ele vation

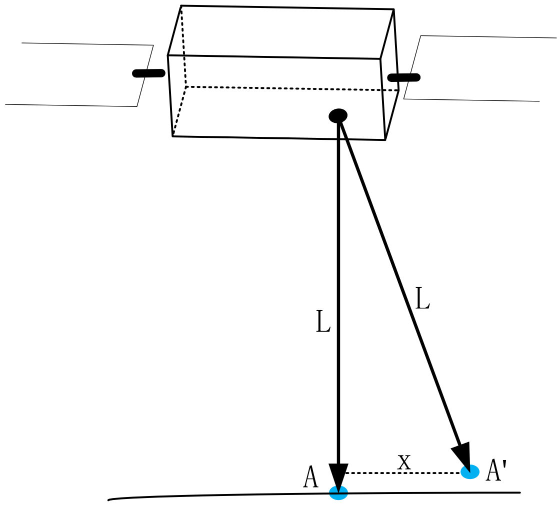

Figure 2 Laser footprint level versus elevation change

In a strict sense, when the horizontal coordinates of a laser footprint change, the elevation data is supposed to change accordingly. As shown in Figure 2, a satellite is assumed to emit a laser ranging value of L perpendicular to the surface of earth, with the laser footprint denoted as Point A. If the laser footprint moves X meters in a certain direction to Position A', the formula used to calculate the elevation difference Δh between the two points is expressed as Equation (1).

$$\Delta h = L - \sqrt{L^2 - X^2} \quad (1)$$

where L represents the laser ranging value, and X refers to the offset of the laser footprint.

Usually, active laser altimetry satellites operate in the orbits of around 500 km in altitude, emitting lasers at an angle basically perpendicular to the surface of earth. Typically, a wavelength of 532 nm or 1064 nm is used. Therefore, it is assumed that the laser ranging value from the satellite is 500 km. Figure 3 shows the relationship as determined in this condition between the horizontal offset of the laser footprint and the changes in elevation value.

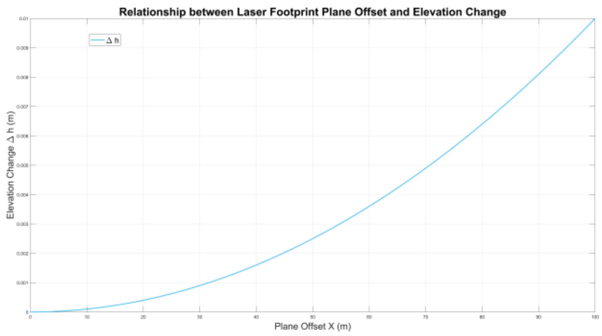

Figure 3 Relationship between laser footprint plane offset and elevation change.

From Figure 3, it can be observed that the change in elevation value before and after the shift of laser footprint is of nonlinear nature, showing a sharp rise in growth rate with an increase in horizontal offset of the laser footprint. However, it is noteworthy that the increase in elevation is merely 0.01 m when the horizontal offset of the laser footprint reaches 100 m. Apparently, when a laser altimetry satellite is operational, it is very unlikely that such significant displacement occurs to the laser footprint. Therefore, it is presumed that the elevation remains constant when the laser footprint moves over a small area.

3.2 Optimised model of laser footprint plane position without satellite parameter support

According to the above laser footprint level and elevation change model, the elevation value can be considered constant when the laser footprint coordinates move in a limited range. Therefore, the laser footprint grid is established with the initial laser footprint as the centre. Meanwhile, the appropriate distance and step size are taken as the range and interval of the grid. Each point of the laser footprint grid is traversed to calculate the difference between each grid point and the corresponding DSM data elevation value. The position with the least significant difference is treated as the optimal offset position, and the offset of the laser footprint is calculated to determine the optimal position of the laser footprint. The process of implementing the model is detailed below.
As shown in Figure 4, the initial laser footprint A is assumed to have coordinates (x_0, y_0). With this as the center, a grid of size $N \times N$ is constructed. Then, a laser footprint grid coordinate system (LFGCS) is established with the upper-left corner of the grid as the origin, as shown in Figure 4. In this coordinate system, the spacing along the x-axis is denoted as Δx, and the spacing along the y-axis is denoted as Δy. The coordinates of laser footprint A are presented as $(\frac{N+1}{2}, \frac{N+1}{2})$, and the coordinates of point B are presented as $(N-1, N-1)$.

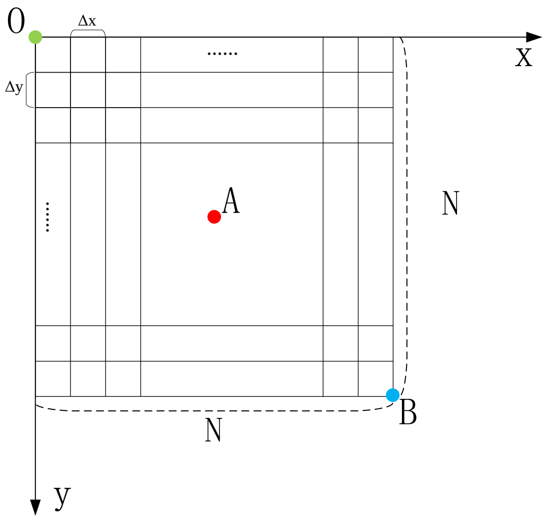

Figure 4 Schematic diagram of the horizontal grid of laser footprints.

Given m laser footprints, the optimal method of calculating the offset point of their plane position is expressed as Equation (2).

$$(x_{opt}, y_{opt})_{\mathrm{LF}} = \arg\ \min_{x,y}\left(\sum_{i=0}^{m-1}\left(\left|h_{Satellite_i}(x_0, y_0) - h_{DSM_i}(x,y)_{\mathrm{LF}}\right|\right)\right) \tag{2}$$

where $(x_{opt}, y_{opt})_{\mathrm{LF}}$ represents the optimal offset position for m laser footprints; $h_{Satellite_i}(x_0, y_0)$ denotes the initial elevation of the i-th laser footprint; $h_{DSM_i}(x,y)_{\mathrm{LF}}$ indicates the DSM elevation value at the coordinates (x, y) within the horizontal coordinate system of the i-th laser footprint; $\arg\ \min_{x,y} f(x,y)$ denotes the set of x, y values that minimizes $f(x,y)$. That is to say, when $(x_{opt}, y_{opt})_{\mathrm{LF}} = (x,y)_{\mathrm{LF}}$, the value of $\left(\sum_{i=0}^{m-1}\left(\left|h_{Satellite_i}(x_0, y_0) - h_{DSM_i}(x,y)_{\mathrm{LF}}\right|\right)\right)$ is minimized.
By calculating the optimal offset position $(x_{opt}, y_{opt})_{\mathrm{LF}}$ of the laser footprint, the displacement $(\Delta x, \Delta y)$ of the laser footprint can be determined according to the positional relationship between the optimal offset and the initial laser footprint location. The formula used to perform this calculation is expressed as follows:

$$\begin{cases} \Delta x = x_{opt_{LF}} - \frac{N-1}{2} \\ \Delta y = y_{opt_{LF}} - \frac{N-1}{2} \end{cases} \tag{3}$$

Therefore, Equations (2) and (3) are combined to obtain the formula used to calculate the coordinates (X, Y) of the laser footprint after model optimization as follows:

$$(X,Y) = (x_0, y_0) + \left(-\frac{N-1}{2}, -\frac{N-1}{2}\right) + \arg\ min_{x,y}\left(\sum_{i=0}^{m-1}\left(\left|h_{Satellite_i}(x_0, y_0) - h_{DSM_i}(x,y)_{\mathrm{LF}}\right|\right)\right) \tag{4}$$

Herein, Equation (4) is the formula used in the laser footprint horizontal position optimization model that requires satellite parameter support. According to this model, only the coordinates of the laser footprint and corresponding high-precision DSM data are required to optimize and correct the position of the laser footprint, which removes the need for support from satellite parameters.

4. Experimental results and discussion

4.1 Experimental results

The subject of this study is a piece of calibrated GF-7 Beam2 laser data. The overall process of the experiment is illustrated in Figure 5. During operation, noise was introduced to the calibrated GF-7 laser data to simulate the horizontal positional deviations of its laser footprint. This simulation is considered necessary as the parameters of the spaceborne laser altimeter are significantly affected during operation and its pointing parameters have a particularly significant impact. Consequently, for the laser data collected from the same orbit, the deviations in the laser footprint are highly consistent. That is to say, the shifts are similar in direction and magnitude.

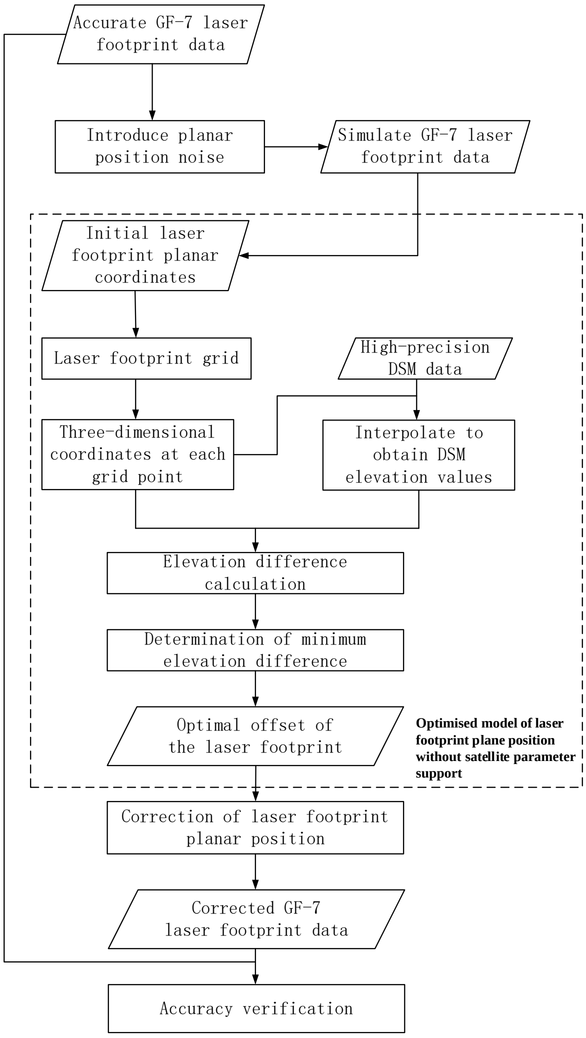

Figure 5 Overall experimental process.

As shown in Figure 6, a random noise error with a mean of 12.56 m was introduced during the experiment into the GF-7 laser data for simulation of the potential offset errors that may occur during satellite operation.

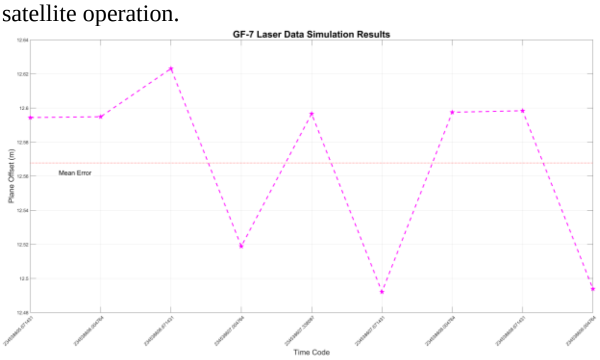

Figure 6 Horizontal position offset of GF-7 Beam 2 laser data after the introduction of noise.

Furthermore, the high-precision DSM data collected from the Xinjiang region was taken as the reference terrain data to correct the horizontal position of the noise-introduced GF-7 Beam2 laser data, using a laser footprint horizontal position optimization model that required no satellite parameter support. Since the GF-7 Beam2 laser data without any noise introduced had been finely calibrated using a ground-based laser detector, the horizontal position of this data was taken as the true laser horizontal coordinates to validate the optimized laser footprint horizontal position. The results of validation are shown in Figure 6. According to Table 2, the horizontal position of the GF-7 Beam2 laser data is improved from 12.56 m to 3.11 m after optimization by the method proposed in this paper. Meanwhile, the 9.45 m horizontal error is eliminated and accuracy is enhanced by 75.23%. The optimization of the model is detailed in Table 2.

Table 2 Optimization in the horizontal position of GF-7 Beam2

Time Code	Offset Eliminated After Optimization
234538605.671	9.50 m
234538606.005	9.50 m
234538606.671	9.58 m
234538607.005	9.38 m
234538607.338	9.50 m
234538607.671	9.31 m
234538608.005	9.50 m
234538608.671	9.51 m
234538609.005	9.31 m
Mean	9.45 m

4.2 Discussion

In the model of laser footprint horizontal and elevation changes, we considered laser altimetry satellites with nearly vertical observations. However, there is a certain angle at which earth is observed by some laser altimetry satellites, although this offset angle is usually in no excess of 5 degrees. For these satellites observing earth at a specific angle, it might be necessary to further consider the changes in horizontal position and elevation of the laser footprint. Figure 7 shows the changes in horizontal position and elevation of the laser footprint when the laser emission angle is considered.

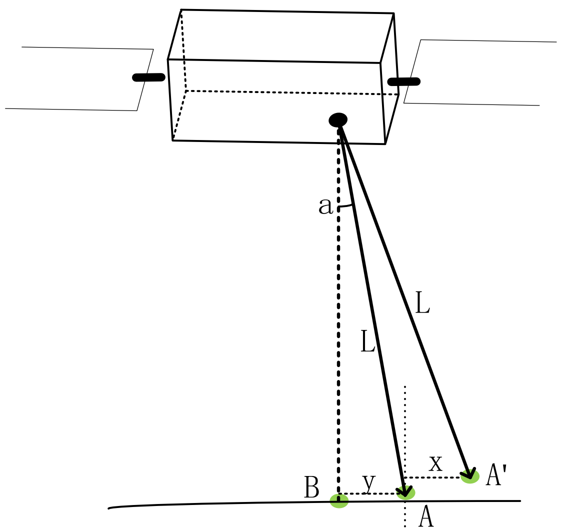

Figure 7 Consideration given to the changes in horizontal position and elevation of laser footprint for laser altimeters during large-angle earth observations.

In Figure 7, A represents the initial position of the laser footprint, A' indicates the position of the laser footprint during the traversal process, B refers to the nadir point of the laser altimetry satellite, α denotes the angle of deviation of the laser from the nadir direction, L stands for the laser ranging value, y indicates the initial distance of the laser footprint from the nadir point, and x represents the shift distance of the laser footprint during the traversal process. According to the geometric relationship shown by the changes in horizontal position and elevation of the laser footprint, a model is obtained as follows for the changes in horizontal position and elevation of the laser footprint after the laser emission angle is taken into account:

$$\Delta h = \sqrt{L^2 - (L * \sin\alpha)^2} - \sqrt{L^2 - (L * \sin\alpha + X)^2} \quad (5)$$

Figure 8 shows the changes in horizontal position and elevation of the laser footprint as observed when the laser ranging value is assumed to be close to the orbital altitude of approximately 500 km, with a deviation angle of 5° from nadir.

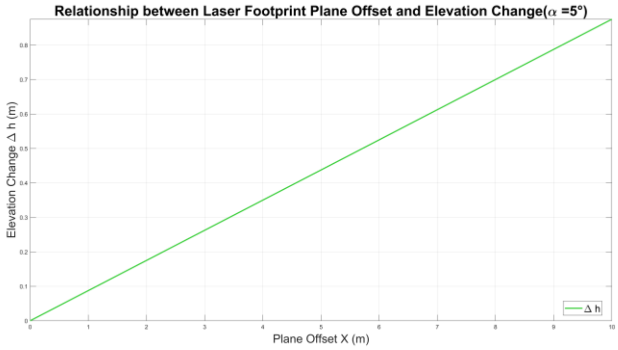

Figure 8 Changes in horizontal position and elevation of the laser footprint given a 5° deviation from Nadir.

According to Figure 8, the elevation of the laser footprint changes significantly with the changes in its horizontal position, given a deviation of 5° from nadir. It is indicated that the model and algorithm proposed in this study are not applicable to the precision optimization of laser footprints from large-angle spaceborne laser altimeters. Therefore, our future research will be focused on how to optimize the footprint position of large-angle spaceborne laser altimeters.

5. Conclusion

In this paper, a study is conducted on the relationship between the changes in the horizontal position of laser footprints and elevation values under the condition that earth is observed at a large angle or almost vertically. In addition, a method of optimizing the horizontal position of laser footprints without the need for satellite parameters is proposed. It is effective in further optimizing laser footprint coordinates without the need for satellite parameters, especially the pointing and ranging correction parameters of onboard laser instruments. With the proposed method applied to GF-7 Beam2 laser data, the accuracy of the horizontal position is improved significantly from 12.56 m to 3.11 m. Meanwhile, a 9.45 m horizontal error is eliminated and the accuracy of horizontal positioning is enhanced by 75.23%. This meets the accuracy requirements for laser data. Additionally, this method is applicable to address the challenge in optimizing laser footprint data without satellite parameters. This contributes an effective solution to the precise positioning of laser footprints. However, no experiment is conducted using real laser footprint data as this study focuses on simulated laser footprint data, which will be discussed in our future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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