

## TOWARDS ENERGY ATLAS OF SOFIA CITY IN BULGARIA

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### ABSTRACT:

This paper proposes the first energy atlas of Sofia in Bulgaria. The research uses a geographical information system (GIS) approach and a statistical tolerance methodology to estimate building energy consumption. The buildings were classified into ten categories, and tolerance intervals were computed, which provide a distribution-free summary for the consumption in each class, suitable for spatial visualisation. GIS is used to classify and visualise the results. The results show a clear contrast in the energy consumption between buildings in highly urbanised areas and those in the suburbs. It was found that the high energy consumption belongs to the areas where the shopping, commercial, industrial and sports buildings are located and already developed. The energy consumption bounds were used to enrich a semantic 3D city model of Sofia. This model can be used for further analysis of energy supply, climate change, urban heat islands, and urban health as well as for calculating the climate scenarios. An extensive outline of the utility and directions for future development of the atlas are provided.

### 1. INTRODUCTION

Nowadays, countries worldwide experience rapid economic growth and face serious energy shortages, especially in terms of electricity. A major concern is the growing energy consumption caused by the rise in global population and the need for energy supply in urban areas (Sheng et al. 2017). Such areas have already felt the effect of climate change and the reduction of green spaces due to high energy consumption, rapid urbanization, and land-use changes. It is well-known that climate change and increasing urban temperatures significantly impact the energy consumption of buildings during the summer and encourage the occupants to use more air conditioning to increase their thermal comfort. This situation leads to the increasing energy demand and further growth of CO<sub>2</sub> emissions. Thus, efficient energy planning through the development of energy atlases with a high spatial resolution is required to understand the current energy situation at a city level and to improve energy efficiency at the neighbourhood scale.

In Sweden, an energy atlas of the multifamily building stock already exists (Johansson et al. 2017). In Turin, Italy, the energy use model has been improved by considering applications to urban areas of different dimensions. An urban energy atlas for the building stock has been defined with the support of a geographical information system (Mutani and Todeschi, 2019). Moreover, the building analysis for urban energy planning using key indicators on virtual 3D city models through the energy atlas of Berlin was developed (Krüger and Kolbe, 2012). Putra and Van Der Knaap (2019) introduced a project related to the energy atlas of Amsterdam. The project aimed to help the city accelerate its energy transition to reduce CO<sub>2</sub> emissions and deal with climate-related issues. A heat atlas was developed in Denmark, to serve as a support tool for energy system models (Petrovic and Karlsson, 2014). Energy system analysis tools incorporate environmental, economic, energy and engineering analyses of future energy systems while considering the assessment of transitional scenarios towards achieving a fossil-free society after 2050.

In much the same way as other cities, Sofia in Bulgaria is experiencing rapid urbanisation and has thus seen an increase in energy consumption. However, there is a lack of studies and comprehensive methodologies focusing on estimating energy consumption at the building level in Sofia, which could be essential for the reduction of both energy consumption and CO<sub>2</sub> emissions. To the best of our knowledge, there has been no attempt to develop an energy atlas for Sofia, or, indeed, for any city in Bulgaria, as of yet.

To address this issue, this research proposes the use of a combination of geographical information system (GIS) methods and tolerance statistics to estimate building energy consumption and develop the first energy atlas in Sofia. The energy atlas of Sofia shows the energy consumption of building types, using available energy efficiency data about individual buildings. It is an essential tool for researchers, society and local governments in coordinating climate action and energy consumption reduction, as well as for setting zero-emission goals and helping citizens and communities by providing useful energy information (Figure 1).

The paper is organised as follows. The next section introduces the data and selected methodology used in constructing the atlas. It reviews tolerance intervals to a level sufficient for the paper. Section 3 provides the results of the tolerance analysis and discusses the development of both 2D and 3D visualisations of these results. Finally, conclusions and a detailed plan for future work are presented in the Conclusion section.

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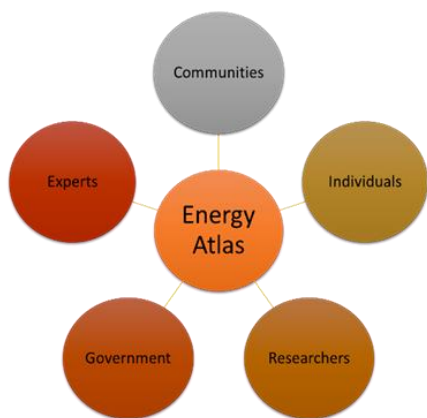


Figure 1. Applications and benefits of energy atlas.

## 2. DATA AND METHODOLOGY

This section describes the input dataset, and the process of calculation of the energy atlas and provides an overview of tolerance statistics.

### 2.1 Study area

Sofia has a population of 1,221,785 people (National Statistical Institute, 2020) and a city area of 492 km<sup>2</sup>. The city is located in the western part of Bulgaria in Sofia Valley, surrounded by the Vitosha mountain to the south and the Balkan Mountains to the north. The location of the city is 42.70° N, 23.33° E, and the city has an average altitude of 550m (Figure 2).

According to the population and housing census, in Sofia, there are 607,473 dwellings and 101,696 buildings. Between 2000 and 2011, 102,623 dwellings were constructed. Sofia's architecture combines a wide range of architectural styles that vary from Christian Roman architecture to the Socialist-era apartment blocks. Along with the increase in the number and density of buildings and population in the capital, the rise of energy consumption has become a main topic and a problem because of the associated change in the local climate, manifested most severely in the increased effect of the urban heat island, deteriorated air quality and heightened carbon emissions.

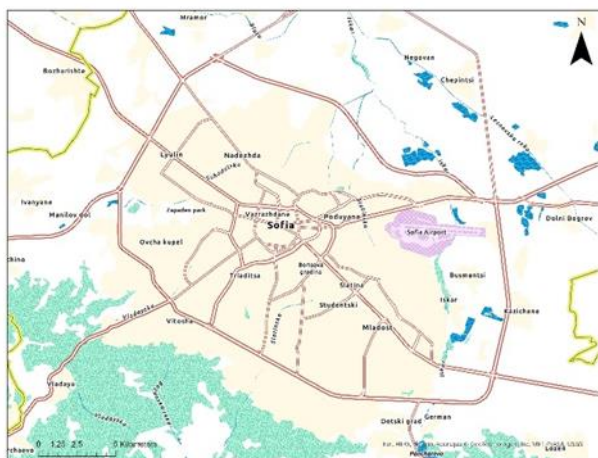


Figure 2. Study area.

### 2.2 Development of the energy atlas

Smart use of energy in homes and businesses has great benefits and results in reduced energy consumption, cost and improves

the negative consequences of climate change in Sofia. By properly combining various energy efficiency measures, energy savings could be achieved by setting zero emission goals. The application of measures and technologies to reduce energy consumption gives several benefits, e.g., healthier working conditions, improved thermal comfort, reduced heat release from the buildings, positive impact on the local climate, environment, and biodiversity, etc.

The study uses a methodology based on GIS and tolerance statistics, to estimate building energy consumption and develop the atlas. The development approach of the energy atlas includes five steps: 1) data collection, 2) GIS classification, 3) tolerance analysis, 4) creation of the energy consumption scenarios and 5) development and visualisation of the energy atlas (Figure 3).

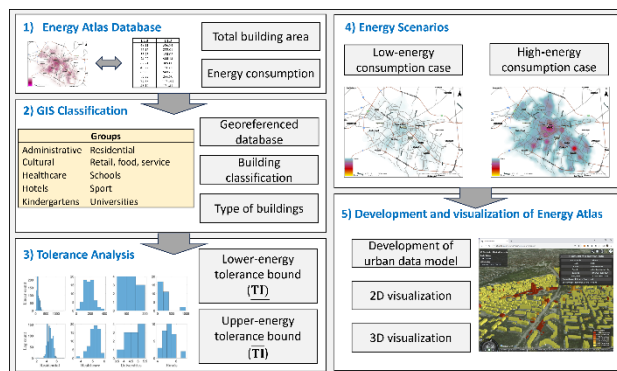


Figure 3. Process diagram of the study.

#### 2.2.1 Data collection

The calculation of the energy consumption is based on the “Building certificates of energy characteristics” dataset from more than 2,500 representative buildings in Sofia (Agency for Sustainable Energy Development, 2023). The energy atlas database includes the total building area and energy consumption per unit area (energy flux).

#### 2.2.2 GIS classification

The buildings are classified into ten categories: residential, healthcare, cultural, administrative, hotels, services and restaurants, kindergartens, schools, universities, and sports, as seen in Table 1.

#### 2.2.3 Tolerance analysis

For each of the categories, a distribution-free ( $p$ ,  $\alpha$ ) tolerance interval, defined as  $TI = [TI, \overline{TI}]$ , (Meeker and Hahn, 2011) was computed, which contains a proportion  $p$  of the population with  $100(1-\alpha)\%$  confidence. The bounds of the tolerance interval can be propagated through further analysis and visualisation. Computing a tolerance interval was chosen, rather than a central tendency statistic, for two main reasons.

Firstly, anybody that is likely to use the atlas will be interested in problematic areas and building types, as indicated by the upper bound of the interval,  $\overline{TI}$ , rather than the average consumption of buildings. This is to say, energy consumption stakeholders will likely not be indifferent to whether individual buildings exceed mandated reference values or not. At the same time the lower bound of the tolerance interval,  $TI$  provides a “best-case” consumption indicative of the state of the art in achievable building efficiency. Central statistics tend to mask such behaviour by only caring about some average measure of consumption for the whole class. The use of a high proportion,  $p$ ,

widens the apparent consumption ranges but provides a statistical guarantee that the unobserved part of the respective building class will not significantly exceed the calculated figures.

The second reason behind preferring the distribution-free ( $p, \alpha$ ) TI to a central statistic is that the nature of the data precludes the usefulness of central tendency statistics due to either complex distributional shapes or small sample sizes. The histograms of 4 of the 10 classes and their log transforms are shown in Figure 4. The TI and tolerance statistics in general can automatically alert the analyst about the achievable confidence, given the size of the data set. Furthermore, assuming a distributional shape for the data will produce tighter results, but will often be unjustified, especially at small sample sizes.

It must be noted here that tolerance intervals provide statistical rather than mathematical confidence and as such if propagated through all but the simplest subsequent analyses their values may no longer possess the same level of confidence or population bounding, even with the use of purpose-made intrusive (Moore, 2009) or non-intrusive (Ioannou, 2023) techniques. Instead, advanced uncertain numbers, such as confidence boxes, translated to the tolerance domain need to be used to obtain these guarantees (Ferson, et al., 2013).

The results of the tolerance analysis are presented in Section 3.1.

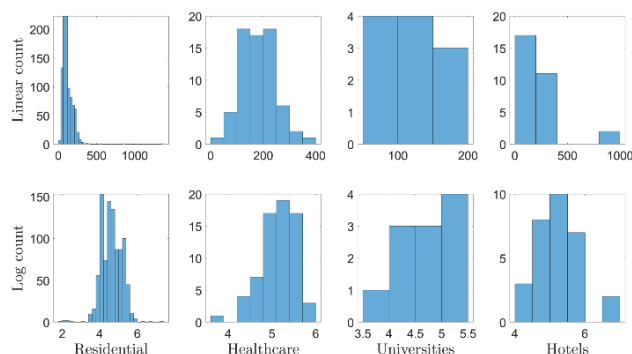


Figure 4. Distribution of data for four example classes. Lognormality cannot be easily supported.

### 2.2.4 Creation of the energy consumption scenarios

Based on the computed tolerance intervals, this study considers two main cases: low (using  $\underline{TI}$ ) and high energy consumption (using  $\overline{TI}$ ) scenarios. Here, GIS is applied to estimate and visualise the final energy consumption of buildings for the whole city and then focus on some specific areas represented by high energy consumption. The map-based visualisation is presented and discussed in Section 3.2.

### 2.2.5 Development of visualisation of the energy atlas

The final step of the work is to present the results in a convenient tool to be used by different stakeholders. The energy consumption scenarios are used to enrich a 3D model of the city, which can, in turn, be visualised and queried in a web browser to provide insights for citizens and support decision-making for city authorities. These results are presented in Section 3.3.

## 3. RESULTS AND DISCUSSION

This section considers the estimation of tolerance statistics, energy scenarios, distribution of the gross floor area, visualisation of the energy atlas of the whole city and by a specific area, and the enrichment of a 3D model of a district of Sofia.

### 3.1 Tolerance analysis for Sofia

All tolerance intervals were computed directly from the data, without assuming a particular distributional shape for any of the building classes. Even though many of the classes suggest a possible log-normal distribution (Figure 4), this hypothesis cannot be definitively accepted across all data with high confidence. Data for all but the Residential and Retail, food, and service classes passed the Kolmogorov-Smirnov test for lognormality at the 5% confidence level (Foreman et al. 2014). However, it is well-known that this test can falsely fail to reject the null hypothesis (log-normality in this case) due to the small sample size. Using distribution-free procedures allows one to relax unreasonable assumptions about the data whilst obtaining the desired coverage probability for the intervals.

Computing tolerance intervals for high population proportions with high confidence levels, as in the present case, is desirable from an application point of view but may be impossible, because certain classes may not have a sufficient number of samples to support the computation. For all categories, except Hotels, Sports and Universities, the data was sufficient to compute the tolerance intervals that contain, at least 90% of the buildings in the category with at least 95% confidence. The results are shown in Table 1. For the Hotels class (28 buildings), Sport class (29 buildings) the Universities class (15 buildings), one can either compute the 90% tolerance bound with lower confidence (about 45% at the lowest for Universities) or preserve the confidence in the estimate but reduce the bound (to 70% at the lowest in this case, again for Universities). Here, the latter option is chosen as confidence is favoured over completeness. This choice is purely volitional and is up to the analyst performing the study.

Group	p	1- $\alpha$	$\underline{TI}$	$\overline{TI}$
Administrative	0.9	0.95	48.11	586.94
Cultural	0.9	0.95	75.15	822.01
Healthcare	0.9	0.95	53.78	354.08
Hotels	0.8	0.95	74.29	944.14
Kindergartens	0.9	0.95	62.71	309.00
Residential	0.9	0.95	46.27	156.68
Retail, food, service	0.9	0.95	63.00	943.62
Schools	0.9	0.95	56.88	266.90
Sport	0.8	0.95	75.38	1171.61
Universities	0.7	0.95	53.15	171.51

Table 1. Tolerance intervals (TI) for the energy consumption (kWh/m<sup>2</sup>/year) of the 10 classes of buildings.  $\underline{TI}$  and  $\overline{TI}$  are the lower and upper tolerance interval values, respectively.

From Table 1 it becomes obvious that the tolerance intervals for different classes overlap heavily, with the interval for the Sports class nearly containing all other intervals. Despite the fact results are based on the data alone, without any unwarranted assumptions, the outcome is unfavourable from an inferential and decisional point of view. The chief reason for this overlapping is the fact that the data has been split only based on building usage and not on other physical features, which could prove more important in refining the results. It is well known that building energy consumption depends strongly on the building type and construction material, thermal envelope (the physical separator between the interior and exterior of the building), the number of occupants and their activities, energy-consuming appliances and

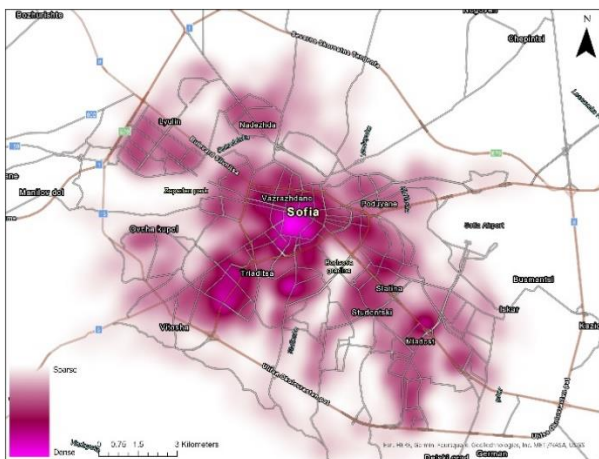
devices in use, and the weather conditions. Such refined analyses are considered part of the future work in developing the atlas, as outlined in Section 4.

### 3.2 Energy scenarios

Two energy scenarios are presented in this section. The first corresponds to the lower bound of the computed tolerance interval and the second to its upper bound. The resulting horizontal distribution of energy consumption is presented visually both for the whole city and for several areas with increased energy use.

#### 3.2.1 The role of gross floor area

Energy flux data is convenient to use because it is portable from the individual building to the class of buildings. However, the result of the presented analysis is to obtain absolute consumption values. To transform consumption TI, computed on energy flux data to such absolute values, each TI must be multiplied by the gross floor area of the building of interest. Gross floor area is calculated as the building footprint area multiplied by the number of storeys of that building. Information about these building features is available from the Geodesy, cartography, and cadastre agency in Bulgaria. The horizontal distribution of the gross floor area is shown in Figure 5.



**Figure 5.** Distribution of total floor area in Sofia. Normalized values represent the total floor area. The pink colour shows the locations with a large total floor area.

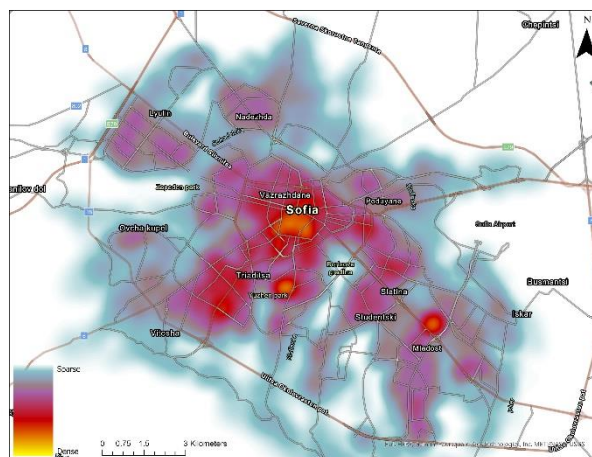
The results show several areas with predominantly large total floor areas in the central, south, and southeastern parts of the city. These areas are mainly represented by non-residential buildings, such as hotels, sports facilities, shopping and entertainment centres, administrative, and cultural buildings. Some of the areas are part of new developments.

#### 3.2.2 Energy atlas of the whole city

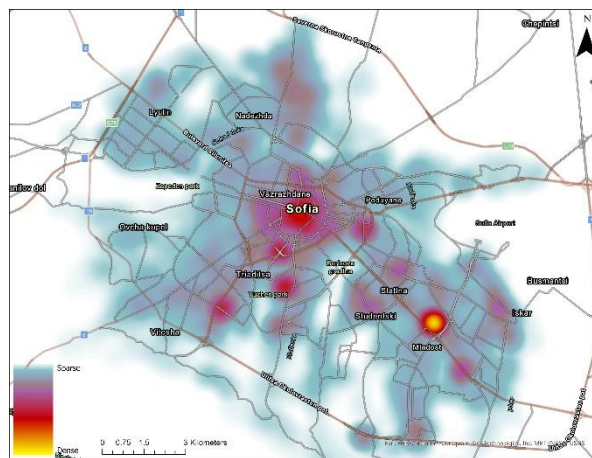
In this study, GIS is applied to estimate and visualise both tolerance bounds of energy consumption of buildings for the whole city and then focus on some specific areas represented by high energy consumption. In this case, the calculation from energy flux to absolute energy is a single multiplication, which preserves the coverage properties of the TI computed in Section 3.1 and shown in Table 1. The energy consumption is represented by normalized values with a specific range depending on the calculation of the lower and upper tolerance bounds for the different classes to emphasise problematic areas.

Figure 6 shows the horizontal distribution of the energy according to the lower bound of the tolerance intervals. There are several areas with energy consumption increases (central, south and southeastern parts of Sofia) among lower tolerance bounds for the different classes, which becomes obvious when the data is visualised. The interesting point here is when we use the small variation, the areas with increased energy consumption overflow with those from the suburbs.

Similarly, Figure 7 shows the horizontal distribution of consumption corresponding to the upper bounds of the tolerance intervals. The results in Table 1 suggest that there will be a more pronounced difference than in the TI case, which is confirmed visually by Figure 7. There is a clear contrast in energy consumption between buildings in highly urbanised areas, especially shopping, commercial, industrial and sports buildings, compared to those in the suburbs. This effect is due to two reasons. The first is that there are clusters of building classes with higher energy consumption. The second reason is that the highlighted areas of the city also contain many buildings with large gross floor area (either high-rise buildings or large area warehouse-type stores).



**Figure 6.** Horizontal final low energy consumption distribution in Sofia. The low energy consumption is represented by normalized values (0 to 0.2). Red and yellow colours show the locations with an increase in energy consumption.



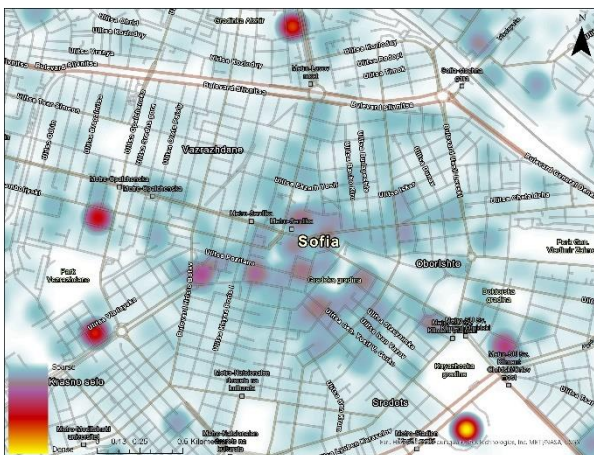
**Figure 7.** Horizontal final high energy consumption distribution in Sofia. The high energy consumption is represented by normalized values (0 to 1). Red and yellow colours show the locations with high energy consumption.

Based on the results of the high energy consumption case in Sofia, three specific areas with high energy consumption are selected and described in detail in the next section.

### 3.2.3 Energy atlas by specific area

The central part of Sofia is home to the buildings of the National Assembly, the Presidency, the Council of Ministers and other governmental institutions. Here, there are also several cultural buildings, sports facilities, hotels and commercial centres and old-type residential buildings.

Figure 8 shows the horizontal distribution of the high-energy consumption spots in central Sofia. It is considered that the main reasons for the energy consumption increase is because of the age and materials used which affect the building envelope (Balaras et al. 2005; Jo et al. 2022). For older buildings, the envelope is generally not as efficient as for newer buildings due to the use of available materials at the time of construction (e.g., no thermal insulation and single-pane windows). There are also losses associated with the ageing of the building (Hauashdh et al. 2022), whereby construction materials lose energy efficiency. This means older buildings naturally require more energy per unit area to be heated and, when necessary, cooled down. Moreover, the presence of restaurants, shops, and other businesses in residential buildings significantly influences their energy consumption.



**Figure 8.** Horizontal final high energy consumption distribution of the central part of Sofia. The high energy consumption is represented by normalized values (0 to 1). Pink, red and yellow colours show the locations with high energy consumption (0 to 1).

The second area identified as having a high energy consumption is located in the southern part of the capital. This area includes part of Bulgaria Blvd. and Lozenets, Ivan Vazov and Strelbishte districts. The area is characterized mainly by residential buildings but also by several large shopping centres, hotels and cultural buildings which contribute to the elevated energy consumption in that area (see red spots in Figure 9).



**Figure 9.** Horizontal high final energy consumption distribution of the south part of Sofia. The high energy consumption is represented by normalized values (0 to 1). Pink and red colours show the locations with high energy consumption (0 to 1).

Additionally, the analysis of the locations of high energy consumption shows a connection with newly and already developed administrative, business, and commercial buildings which are mainly concentrated in the southeastern parts of Sofia near Tsarigradsko Shose Blvd. (Figure 10).



**Figure 10.** Horizontal final high energy consumption distribution of southeastern parts of Sofia. The high energy consumption is represented by normalized values. Pink, red and yellow colours show the locations with high energy consumption (0 to 1).

### 3.3 3D visualisation

The calculated tolerance intervals for building energy consumption are used to enrich the building attributes of the 3D model of Lozenets District (Dimitrov and Petrova-Antonova, 2021). The 3D model is developed in level-of-detail (LOD) 1 following the CityGML 2.0 standard (Gröger et al, 2012). The geometry of the 3D model covers the whole city while building attributes are modelled for the Lozenets District, shown in Figure 11.

The new attributes extend the building module of CityGML. The standard is imported into a 3DCityDB database, which is a geospatial relational database that stores, represents, and manages 3D city models on top of an existing spatial relational database such as PostGIS (Yao et al., 2018). The 3D model is

visualised using Cesium (<https://cesium.com/>) virtual globe allowing user interaction and perception. A web application, hosted on a local Node.js web server, is developed to visualise the city model. Cesium.js is used to implement the web application due to its support of rich functionality such as attributes display and query, object handling, highlighting, and map layer control, among others.



**Figure 11.** 3D City Model visualised in Cesium ion. The coloured buildings belong to the Lozenets District.

The following filtering functionality is implemented and can be invoked by the main menu of the web application:

1. Silhouette a building on mouseover and show its class as overlay content.
2. Silhouette a building on selection and show its attributes, including energy consumption bounds in an information box.
3. Show buildings in different colours depending on their attributes, including energy consumption bounds.
4. Show buildings in transparent mode.
5. Show buildings according to a logical condition.
6. Show shadows depending on the current time.

The current functionality in points 2., 3., and 5. of the web application is extended to support user interaction to show buildings in different colours depending on their  $\underline{Tl}$  and  $\overline{Tl}$  energy attributes (Figures 12 and 13). Thus enriched, the dynamic 3D model can serve users by providing them insight about energy consumption and its relationship with other attributes of the building stock already present in the model.



**Figure 12.** Energy  $\underline{Tl}$  energy visualisation of 3D City model. Red colours show the buildings with high energy consumption in Lozenets District.



**Figure 13.** Energy  $\overline{Tl}$  energy visualisation of 3D City model. Red colours show the buildings with high energy consumption in Lozenets District.

## 4. CONCLUSIONS AND FUTURE WORK

### 4.1 Conclusions

This paper presented a first attempt at developing an energy atlas of Sofia in Bulgaria. The research used a combination of geographical information system approaches and tolerance analysis to estimate building energy consumption. A means of visualising the results in both 2D and 3D manner were tested and validated. The main conclusions of the study are as follows:

1. Classification of the building stock in Sofia, by its usage type, provides a lightweight basis for computation but obscures many nuances of the buildings, important for creating an energy atlas.
2. Based on this classification, the results show several areas exhibiting a clear contrast in energy consumption between buildings in highly urbanised areas and those in the suburbs.
3. Buildings with high energy consumption belong mostly to the retail, food, service and sports classes.

## 4.2 Future work

Based on the preliminary results of creating the energy atlas, there are several important directions for future development. First and foremost, subsequent studies will focus on examining the quality of the data. Missing or censored values, dubious data and measurement processes will be investigated. Secondly, the data will be enriched with features known a priori to be important drivers of energy consumption. These will be instrumental in providing a reliable separation of buildings in Sofia, which in turn is vital for a robust analysis of their energy consumption. Thirdly, despite the sample size not being an immediate issue, as demonstrated by the use of an appropriate tolerance analysis approach, having more data will enable the use of different, more data-intensive analysis methods from machine learning. This, in combination with a more fine-grained classification and geolocation, will allow a qualitatively different type of analysis to be conducted. Finally, various enhancements to the software visualisations are planned for the future, which will include adding more objects to the 3D model and enabling a wider variety of features to be accessible in Cesium.

The results of the energy atlas are a valuable basis for further analysis of energy supply, climate change, urban heat islands, and urban health as well as for calculating climate scenarios and seasonal effects on urban climate, all of which are steps planned for the future.

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