A FUZZY MEASURE THEORY BASED PROBE INTO A SPATIAL VARIATON ON SURFACE TEMPERATURE OVER INDIA

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KEY WORDS: Dempster-Shafer Theory; fuzzy set; joint belief measure; northeast India; surface temperature

ABSTRACT:

This study used time series and fuzzy measures to comprehend the surface temperature over Northeast India (NEI), Northwest India (NWI), and Central India (CI). We presented a technique for calculating the uncertainty surrounding surface temperature data related to the Indian summer monsoon from 1901–2007. We utilized the Dempster-Shafer Theory to produce belief measures once the random variables had been standardized. We used two judging criteria and derived the joint belief measures for all the cases. This approach gave us some insight into the uncertainties associated with the three-time series taken from three spatially separated zones. It was confirmed that the strength of the evidence across a longer time horizon is a more predictable parameter.

1. INTRODUCTION

In recent years, soft computing techniques like fuzzy set theory and fuzzy logic have gained popularity because they are more flexible than traditional set theory and propositional logic (Narasimha Murthy et al. 2018). It can be seen as a logical structure that mimics how individuals understand and reason. Professor LA Zadeh (Zadeh, 1965) developed fuzzy set theory and fuzzy logic as a result of his research on "fuzzy sets." A way to representing uncertainty that is more broadly applicable is the Dempster-Shafer Theory (DST). A generalisation of the Bayesian theory of subjective probability, DST is also known as the idea of belief functions (Xiao, 2020; Sentz et al. 2002; Denoeux, 2000; Chen et al. 2012; Dubois et al. 1998). Belief functions help us create levels of belief for one topic depending on probability for another, just like in Bayesian theory (Kohlas et al. 2013; Dezert et al. 2012; Watanabe et al. 2018; Bernardo et al. 2009; Ghosh et al. 2006; Weise et al. 1993; Damien et al. 2013). Depending on how closely the two concerns are related, the mathematical characteristics of probability may or may not be present in these degrees of confidence. Despite the fact that Dempster and Shafer's research on the Dempster-Shafer theory was conducted in 1968 and 1976, respectively, the logic it uses is rooted in seventeenth-century thought. This concept was discovered by AI researchers who were searching for a way to apply probability theory to expert systems in the early 1980s (Karni ,2007; Rouder et al. 2005). Although recent research has revealed that controlling uncertainty calls for more complexity than is possible in straightforward rule-based systems, the Dempster-Shafer theory is still relevant due to its relative adaptability (Neapolitan et al. 1990). The Dempster-Shafer theory (Zadeh, 1983) is built on two ideas: deriving degrees of belief for a specific issue from subjective probability for a connected matter; and combining such degrees of belief when they are backed by various sorts of evidence using Dempster's rule. In most cases, degrees of belief are calculated using probability for one question and another. The core principle of Dempster's rule is the notion that the questions for which we have probabilities are independent in terms of our subjective probability assessments. Nevertheless, this independence is merely a priori; it vanishes anytime there is a discrepancy in the data. Before employing the Dempster-Shafer theory to a particular problem, it is necessary to solve two connected problems. Prior to assembling any particular pieces of evidence, we must first classify the problem's uncertainty. Using computational methods, Dempster's rule is used in the next stage. The remedies to these two issues are interconnected as well. Beynon (Beynon et al. 2000) contends that the DS theory of evidence has the potential to greatly outperform "traditional" decision analysis techniques. Dempster's work on probabilities with upper and lower bounds served as the foundation for the DS approaches. They have now gained widespread acceptance in the literature on Artificial Intelligence (AI) and Expert Systems because to their emphasis on assimilating data from multiple sources. In this article, the fundamental ideas behind the DST of evidence are described, together with a brief overview of its background in history and similarities to the more conventional Bayesian theory. The important analytical and applicational issues and most recent developments in this theory were then reviewed. A DST and Analytic Hierarchy Process (AHP) example was used to examine probable future advancements. In the 1980s, Zadeh (Zadeh, 1986) wrote a ground-breaking study demonstrating how the DST of evidence had enormous promise for the field of artificial intelligence in tackling ambiguity in expert systems. In the context of relational databases, second-order relations in first normal form can be retrieved using conventional retrieval methods, according to Zadeh. The relational technique helps the implementation of the Dempster-Shafer theory in AI-related applications and addresses some of its problems.

A new complete surface temperature data set for India is used to visualise temperature changes over the course of seven decades in order to look into the trends and potential effects of global warming. For the purpose of examining the temperature trends at each of these phases, the data set is divided into three categories: pre-monsoon, summer

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https://doi.org/10.5194/isprs-archives-XLVIII-1-W2-2023-707-2023 |

monsoon, and post-monsoon. The previous few decades have seen a rise in meteorologists' concerns regarding global warming. (Kothawale et al. 2010) made the discovery of the Kothawale and Weakened Seasonal Asymmetry of Temperature Trends, which was thought to be the result of a spike in temperature during the monsoon. (Ross et al. 2018) the multi-decadal patterns of surface temperature change over India were studied in a recent study. They discovered that while northeastern and southwest India cooled, northwest and southern India warmed. The importance of periodically checking the ambient temperature is well understood (Dash et al. 2007, Kothawale et al. 2010, Sonali et al. 2013). (Dash et al. 2007) showed a decrease in the minimum temperature over India during the summer monsoon and reported an increase in the months after the monsoon. They (Dash et al. 2007) have demonstrated how seasonal asymmetry and variations in atmospheric circulation may result from a significant variation in seasonal temperature anomalies. According to (Kothawale et al. 2010), although there are daily variations in pre-monsoon maximum and minimum temperatures over specific meteorological regions of India, the size of these variations has not changed significantly from day to day. Reference (Sonali et al. 2013) conducted a spatial and temporal trend analysis of temperature time series at different scales for both lowest and maximum temperatures. They used a sequential Mann-Kendal test to uncover the trend pattern at yearly or seasonal levels following a trend detection analysis using various non-parametric approaches taking serial correlation into consideration. Reference (Smith et al. 2007, Dai et al. 2015) stated that despite a continuous rise in atmospheric greenhouse gases since roughly 2000, the global-mean surface temperature has shown no visible warming, which is a stark contradiction to model forecasts. Using statistical methods and satellite data, Reference (Kaufmann et al. 2003) examined how surface temperatures in North America and Eurasia were affected by inter-annual fluctuations in vegetation within land covers. (Nag Ray et al. 2021) discussed a two-state Markov chain method and an autoregressive method for analysing surface temperature time series over northeast India. Surface temperature is crucial for modelling urban climate, as illustrated in references (Maimaitiyiming et al. 2014, Barrows et al. 2007, Devi et al. 2022, Devi et al. 2023).

In the study described in the paper, based on time series and fuzzy measures, we have concentrated on interpreting the surface temperature corresponding to the summer monsoon (JJAS) over Northeast India (NEI), Northwest India (NWI), and Central India throughout 1901–2007. We have considered 3 random variables X_1, X_2, X_3 representing the temperature of JJAS period over NEI, NWI and Central India respectively. The realisations of the 3 random variables constitute the 3-time series having entries from the data source as explained above. In the following sections, we shall outline a technique to determine the level of uncertainty surrounding surface temperature data related to the summer monsoon in India. We shall standardize the random variables, and the Dempster- Shafer Theory (DST) (Zadeh, 1983) will be used to produce common goals. We will employ two fuzzy numbers representing the two criteria for this purpose. We will present three polynomials to demonstrate a functional link between time series and the measure of joint belief. The uniqueness of the current study lies in the fact that, rather than presenting the traditional statistical methodology, we will examine the data uncertainty using DST (Shafer, 1992). Specifically, we will consider two separate queries on the same universe of discourse and use Dempster's rule of combination to find the join belief measure to obtain the combined belief measure.

2. METHODOLOGY

2.1 Data

Here, we provide a brief overview of the study's data exploration. The Indian Institute of Tropical Meteorology (IITM), Pune, India, has a website from which surface temperature data are collected.https://tropmet.res.in/static_pages.php?page_id=54 is the URL of the website. The IITM, an Autonomous Institute within the Ministry of Earth Sciences, has archived the data, and Sontakke et al. (2008) provide the related details.

2.2 Belief Measure

The fuzzy measure assigns a value to each crisp set within a universal set signifying the degree of evidence or belief that a particular element belongs to the set. Therefore, if X represents a universal set, a fuzzy measure can be defined as a function from the power set of X to [0,1]. A belief measure is another form of fuzzy measure that satisfies some axioms in addition to the axioms satisfied by fuzzy measure. The brief mathematical overview is presented in the subsequent paragraph.
The axioms of fuzzy measures are as follows:

g_1: $g(\emptyset) = 0$ *and* $g(X) = 1$

g_2: For every $A, B \in \mathcal{P}(X)$, if $A \subseteq B$ then $g(A) \leq g(B)$

g_3: For every sequence $(A_i \in \mathcal{P}(X) | i \in \mathbb{N})$ of subsets of X, if either $A_1 \subseteq A_2 \subseteq \cdots$ or $A_1 \supseteq A_2 \supseteq \cdots$ (i.e., the sequence is monotonic), then $\lim_{i\to\infty} g(A_i) = g \lim_{i\to\infty}(A_i)$

A belief measure (Wang and Klir 1992) is defined by a function

$$Bel: \mathcal{P}(X) \to [0,1] \quad (1)$$

which satisfies the axioms of fuzzy measures and another additional axiom define as follows (Klir and Folger 2015):

$$Bel(A_1 \cup A_2 \cup \ldots \cup A_n) \geq \sum_i Bel(A_i) - \sum_{i<j} Bel\big(A_i \cap A_j\big) + . \ldots + (-1)^{n+1} Bel(A_1 \cap A_2 \cap \ldots \cap A_n) \quad (2)$$

for every $n \in \mathbb{N}$ and every collection of subsets of X.
Every belief measure, as well as its dual plausibility measure, can be described as a function.

$$m: \mathcal{P}(X) \to [0,1]$$

Such that $m(\emptyset) = 0$ and

$$\sum_{A\in \mathcal{P}(X)} m(A) = 1 \quad (3)$$

where $m(A)$ denotes the degree of evidence supporting the theory that a specific member of X belongs to the set A but not to any special subset of A, or the degree to which we believe such a claim is justified.

2.3 Dempster-Shafer Theory

Dempster-Shafer theory (DST) (Zadeh 1986) is a generalised method for describing uncertainty. An expansion of probability theory is how this theory is described. The DST is especially helpful when each piece of data implicates a number of distinct conclusions and the support for each unique conclusion is calculated from the overlapping contributions of different pieces of evidence. To determine a degree of belief, the DST has been applied in target identification applications and tactical inference. It is necessary to establish the theory of evidence before the facts may be assembled. The theory of evidence is so named because it addresses the significance of the evidence.

Assume a collection of *n* mutually exclusive and exhaustive propositions, $\ominus = \{X_1, X_2, X_3 \dots \dots X_n\}$, where 0 is referred to as a discernment frame. Thus, the Boolean operation OR can be used to build propositions; $2^{\ominus}$ is the set of all the subsets of 0. To attribute evidence to a proposition, Dempster–Shafer introduced the concept of mass probability, which is denoted by *m(X)* where, X is the universe of discourse.

$$0 \leq m(X_i) \leq 1$$

$$\sum_{X \subseteq \ominus} m(X) = 1$$

$$m(\emptyset) = 0$$

Basic Probability Assignment (BPA) is another term for mass probability. For a hypothesis the support is the total degree of belief which is to be true. Therefore, we can define belief function as

$Bel: 2^{\ominus} \longrightarrow [0,1]$

$Bel(X) = \sum_{Y \subseteq X} m(Y)$ for each $X \subseteq \ominus$

$Bel(X)$ denotes the degree of support for the proposition of X ,then the multiple hypotheses become

$$Bel(X) = \sum_{X_i \subseteq X} m(X_i)$$

Some characteristics of a belief function are as follows:

$$Bel(\ominus) = 1 \; as \sum_{X \subseteq \ominus} m(X) = 1$$

$$Bel(X) = 0 \; if \; X \subseteq \ominus$$

$$0 < Bel(X) < 1 \; if \; X \subseteq \ominus \; and \; X \neq \ominus$$

$$Bel(X) = m(X) \; for \; each$$

$X \subseteq \ominus$ containing only one element

$$Bel(X) + Bel(\bar{X}) \leq 1$$

$$Bel(\ominus) = Bel(X \cup \bar{X})$$

$$= Bel(X) + Bel(\bar{X}) + \sum_{\substack{X \cap Y \neq 0 \\ \bar{X} \cap Y \neq 0}} m(Y) = 1$$

When belief is not allocated to a certain subset it is termed as a non-belief and is associated with $\ominus$. In most cases, a belief function is stated in terms of BPA. Conventionally, the mass probability of the empty set is zero, i.e., $m(\emptyset) = 0$.

This is the formula for Joint belief i.e., $Bel_{1,2}$ from combined body of evidence:

$$m_{1,2} = \frac{\sum_{B \cap C = \emptyset} m_1(B) \, . \, m_2(C)}{1 - K} \tag{4}$$

$$K = \sum_{B \cap C = \emptyset} m_1(B) \, . \, m_2(C)$$

3. RESULTS AND DISCUSSION

This section analyses the surface temperature data during the summer monsoon (JJAS) in North East India from 1901 to 2007. All the values have been standardized. In the following stage, we considered a universe of discourse for the summer monsoon (JJAS) and computed the mean and median to produce a fuzzy set that is "close to the mean and median respectively." The following transformation is used to scale the data so that its values fall between 0 and 1 if x represents the realization of the time series at a certain time point:

$$z_i = \frac{y_i - min \, y_i}{max \, y_i - min \, y_i} \tag{5}$$

where the suffix i denotes realization at the ith time point. A fuzzy set has been created whose elements are drawn from the time series as members of a crisp set and whose membership function is given by

$$\mu_{(z)}(z) = \frac{1}{1 + (z - c)^2} \tag{6}$$

In order to calculate the joint belief, c will be taken into consideration as the mean and median for the two judging criteria.

The methodology mentioned in the previous section is now being applied in this final stage. In the first scenario, we considered two basic assignments, m_1 and m_2, for the summer monsoon (JJAS) for 107 years, where m_1 and m_2 stand for basic assignments for mean and median, respectively. Then, three focal elements R, D, and C were taken into consideration, each of which represented a fuzzy set "very close" to the mean or median, "close" to the mean or median, and "moderately close" to the mean or median, respectively. Furthermore, we calculated R, D, C, as well as the unions of all the focus elements, i.e., i.e., R ∪ D, R ∪ C, D ∪ C, and R ∪ D ∪ C, which are assigned with the membership grades derived from the various α- cuts. For each focal element, we have determined the joint belief, or "$Bel_{1,2}$" by applying Dempster's rule to m_1 and m_2.Again, we created two data sets for the summer monsoon (JJAS) spanning 50 years each. We determined the focal elements R, D, and C and the unions of all the focal elements and followed the same procedure mentioned in the first case. By using four sets of time periods, each including data for 25 years, we determined the joint belief and basic assignments for all the focal elements of summer monsoon (JJAS).

In the final section of the research, we developed three polynomials as a functional link between the time scale and the joint belief measure, where the belief measure is viewed as an independent variable and it can be written as follows:

$$\boldsymbol{z = a_1 y^3 + a_2 y^2 + a_3 y + a_4}$$

$$\boldsymbol{z = a_1 y^4 + a_2 y^3 + a_3 y^2 + a_4 y + a_5}$$

$$\boldsymbol{z = a_1 y^5 + a_2 y^4 + a_3 y^3 + a_4 y^2 + a_5 y + a_6}$$

In this instance, z is a joint belief measure and y is a timeline. All real coefficients have subsequently produced polynomials of degrees 3, 4, and 5. The polynomials didn't seem to be all monic. The leading coefficient is therefore not equal to 1. We have produced a three-dimensional surface in each example by varying the leading coefficients throughout a range of values. The surface of Fig. 1 features an ascending pattern with a time scale, as can be seen.Additionally, the inclination is steeper when the leading coefficient has higher values.

Because the joint belief measure consistently exhibits an upward trend, this suggests that we are evolving through time. As time goes on, the level of evidence becomes more compelling. Since higher time scales are more predictable than lower time scales, this climate parameter is less predictable at lower time scales. This suggests that nonlinear approaches are essential for more rapid climate parameter prediction.

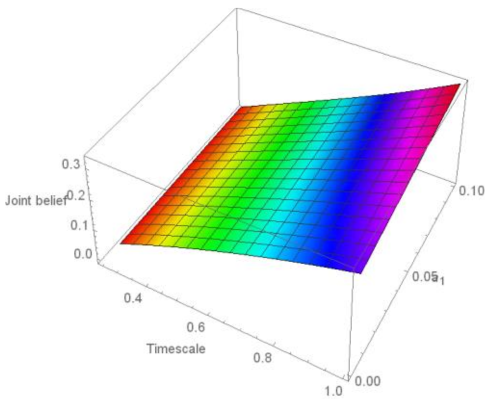

Figure 1: Diagrams illustrating the evolution of the joint belief measure over time for the surface temperature of JJAS over NEI

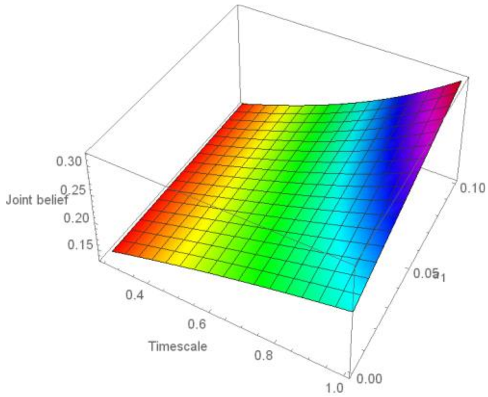

Figure 2: Diagrams illustrating the evolution of the joint belief measure over time for the surface temperature of JJAS over NWI

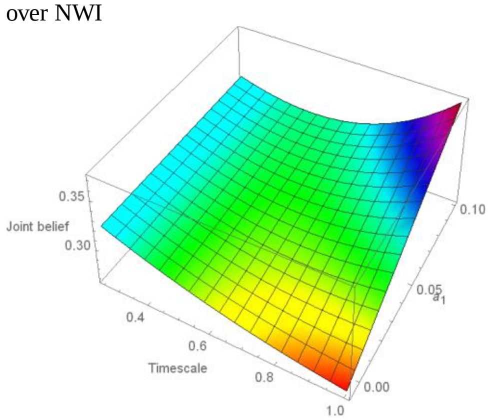

Figure 3: Diagrams illustrating the evolution of the joint belief measure over time for the surface temperature of JJAS over Central India

4. CONCLUSIONS

In this study, we describe how the Dempster-Shafer theory was used to evaluate a time series of surface temperature measurements over a meteorological division of India. According to this theory, two basic assignments are merged and used to describe evidence that was gathered from two separate sources in the same context but related to the same area of study. we have determined R, D, C, and the unions of all focus components, i.e., R ∪ D, R ∪ C, D ∪ C, and R ∪ D ∪ C, assigned with the membership grades derived from the various α - cuts. We have determined the joint belief, or "$Bel_{1,2}$" for each focal element by applying Dempster's method to m_1 and m_2. In the final part of the work, we created three polynomials as a functional relationship between the time scale and the measure of joint belief. This shows that we are evolving through time because the joint belief measure consistently shows an upward trend. As time goes on, the strength of the evidence increases. As a result, this climate parameter is more predictable at longer time intervals than it is at shorter time scales. This shows that nonlinear techniques are essential for forecasting climate factors over shorter timescales. Through this, we expect to move towards extending this kind of DST-based approach to a broader perspective of the multivariate climatological forecast.

While concluding, let us summarize the study's findings. The surface temperature time series over NEI, NWI, and Central India have been investigated in this work. The majority of studies in this direction frequently use trend analysis. Here, the fuzzy measure gives us insight into the inherent complexity of three spatially dispersed time series. Knowing that training and test examples may be created using fuzzy measures, we proposed creating a predictive model for spatiotemporal fluctuations and the training-to-test case ratio created using the fuzzy technique.

ACKNOWLEDGMENTS:

The data utilized in work are taken from the Indian Institute of Tropical Meteorology website (IITM). https://tropmet.res.in/static_pages.php?page_id=54 is the link to the data.

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