TPU Estimation for Bathymetric LiDAR

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Abstract

A tool for Total Propagated Uncertainty (TPU) estimation is presented, as implemented feature of a bathymetric LiDAR processing software. The algorithm of the TPU Estimator is based on an approach alternative to existing methods. The concept of the novel approach is described, compared with and validated against an established tool. Two examples from different coastal surveys document the application of TPU estimation and allow to observe systematic tendencies of the results. We discuss potential for further improvement and consider how to embed TPU estimation into a user interface to generate expressive deliverables as part of hydrographic survey quality verification. Finally, we give an outlook for possible use of LiDAR TPU estimation apart from a-posteriori quality control and outline further development projects.

1. Introduction

1.1 Background

Many hydrographic offices require standardized quality control from surveying service providers. The applicable criteria are defined by the International Hydrographic Organization IHO in Standards for Hydrographic Surveys S-44 (S-44 Edition 6.1.0), wherein the stringency of the criteria corresponds to the degree of critical impact for safety of navigation in five different orders. To assess the compliance of a hydrographic service or product with the relevant order, the measurement equipment's general aptitude as given by specifications resulting from its design, and the conformity of use by qualified personnel is a prerequisite. Initial performance statements and reference measurements allow to state an equipment's capacity to provide the demanded depth performance, bathymetric coverage and feature detection. Further, it is necessary to consider the Total Propagated Uncertainty (TPU) consisting of all contributing uncertainties from system components and environment.

1.2 Total Propagated Uncertainty TPU – Definition

According to IHO Standard S-44, TPU is "Total Propagated Uncertainty (TPU): Three-dimensional uncertainty with all contributing measurement uncertainties included." (S-44 Edition 6.1.0 p. ix) Whereas uncertainty means "Estimate characterizing the range of values within which the true value of a measurement is expected to lie as defined within a particular confidence level." A horizontal (THU; "Total Horizontal Uncertainty") and a vertical (TVU; "Total Vertical Uncertainty") component are defined. (ibid p. ix).

TPU as means of quality control has been in use for acoustic hydrographic datasets for many years. Providing TPU for bathymetric LiDAR allows consistent quality control, using TPU models to independently compare survey results from acoustic and optical instruments, and to account for homogeneous quality for the resulting chart product.

In attributing TPU values to the bathymetric measurement points, complementary survey requirements can be identified for areas where the given values do not reach a threshold defined by the published standard or criteria set by an organization or a contracting authority for a specific use case.

2. Bathymetric LiDAR TPU Estimation

2.1 Concept

A tool for TPU estimation for bathymetric LiDAR, cBLUE, has been developed by Oregon State University (cBLUE, 2025). In parallel, LiDAR manufacturers and working groups are working on different methods for LiDAR TPU estimation, by analytical approaches based on the General Law of Propagation of Variance (GLOPOV) or by a probabilistic and modeling approach based on water surface simulation, Monte Carlo and Ray-tracing principles, or in a hybrid way (Cottin et al, 2020) and (Firat et al, 2019). Building on the preliminary work presented by Antoine Cottin in 2020, we present a new tool, the RIEGL TPU Estimator, which we validate against cBLUE as an established benchmark. The novel concept of the RIEGL TPU Estimator has been first presented at JALBTCX 2024 (Pfennigbauer et al, 2024). Valuable input from discussions at the workshop have led to a series of further development considerations and testing, which we will present in the following.

The RIEGL TPU Estimator relies on uncertainty propagation based on the contributing systematic and random measurement uncertainties from the following sources:

- Uncertainties of the 6 trajectory elements, i.e. the uncertainties $\sigma_{\mathrm{traj,N}}$, $\sigma_{\mathrm{traj,E}}$, $\sigma_{\mathrm{traj,D}}$ of the 3 position coordinates with respect to the North-East-Down frame and the uncertainties σ_{roll}, σ_{pitch}, σ_{yaw} of the 3 attitude angles, which are a by-product of the trajectory calculation.
- Uncertainty σ_{ϕ} of the laser beam's direction (constant value depending on the instrument type)
- Uncertainty σ_r of laser range (instrument property)

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Following this concept, no environmental factors are required as input data, all information for the TPU estimation are being deduced inherently from the LiDAR data. The quality of the grid-based Water Surface Model (WSM), which is crucial for the positions and accuracies of the submerged points, is estimated from the scan data itself: a z-value uncertainty ($\sigma_{z,\mathrm{WSM}}$) and a surface normal vector's angular uncertainty ($\sigma_{\mathbf{n},\mathrm{WSM}}$) are calculated for each grid position. Range uncertainty is estimated from an amplitude-dependent look-up table that results from the calibration of the individual laser scanner device. Angular uncertainty of the laser beam depends on the scan mechanism used and is assumed to be constant.

For uncertainty estimation, we apply a purely analytical approach for TPU estimation based on the Special Law of Propagation of Variance (SLOPOV):

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial i_1}\cdot\sigma_{i_1}\right)^2 + \left(\frac{\partial f}{\partial i_2}\cdot\sigma_{i_2}\right)^2 + \cdots + \left(\frac{\partial f}{\partial i_N}\cdot\sigma_{i_N}\right)^2} \quad (1)$$

where $f(i_k)$ is a function of n input parameters i_k having uncertainties $\sigma_{i_k}$and σ_f is the uncertainty of the resulting function value.

Depending on the measurement point (topographic or bathymetric), two different estimation models are used (see Figure 1 and Figure 2). In both cases, we obtain the coordinate uncertainties σ_x, σ_y and σ_z of the point P as preliminary result. Note, that all uncertainties are assumed to be given as one-sigma uncertainties (corresponding to a confidence level of about 68%).

THU and TVU (both corresponding to a confidence level of 95%) are calculated as follows:

$$\mathrm{THU} = 2.45\cdot\sigma_{xy} = 2.45\cdot\sqrt{\frac{{\sigma_x}^2+{\sigma_y}^2}{2}} = 1.73\cdot\sqrt{{\sigma_x}^2+{\sigma_y}^2} \quad (2)$$

where 2.45 is the confidence radius (95%) of the normalized 2D Gaussian and σ_{xy} is the point's mean isotropic uncertainty within the xy plane calculated from the coordinate uncertainties σ_x and σ_y.

$$\mathrm{TVU} = 1.96\cdot\sigma_z \quad (3)$$

where 1.96 is the symmetric confidence interval (95%) of the normalized 1D Gaussian.

2.1.1 Topographic TPU Estimation Model: In Figure 1 the topographic TPU estimation model is shown. We start from the coordinates of the point P with respect to the Scanner's Own Coordinate System (SOCS). An Auxiliary Coordinate System (AUCS) is introduced which is just rotated with respect to SOCS, pointing its z-axis in laser beam direction (its x- and y-axis may be chosen arbitrarily). Hence, the coordinates of P with respect to AUCS are (0, 0, r), where r is the laser range. Using r, its amplitude-dependent laser range uncertainty σ_r and the laser beam's direction uncertainty σ_ϕ, the coordinate uncertainties of P with respect to AUCS are set to:

$$\boldsymbol{\sigma}_{\mathbf{p}}^{(\mathrm{AUCS})} = \begin{pmatrix} r\cdot\sigma_\phi \\ r\cdot\sigma_\phi \\ \sigma_r \end{pmatrix} \quad (4)$$

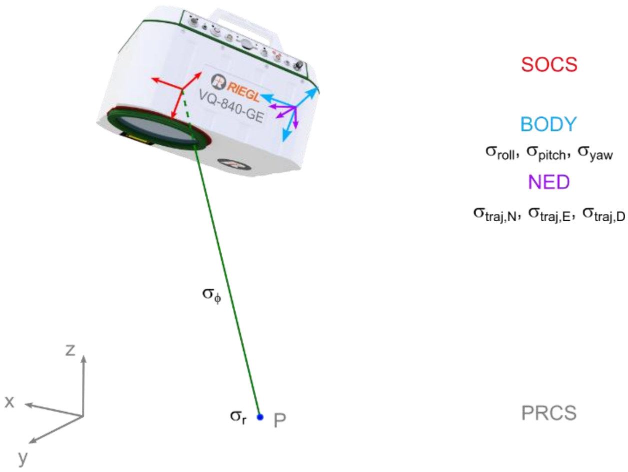

Figure 1. Ray-based model being used for TPU estimation in case of a topographic point.

In order to estimate the coordinate uncertainties of P with respect to PRCS (PRoject Coordinate System), the uncertainties of Equation (4) are propagated using the following transformation chain:

$$\mathbf{p} = \mathbf{q} + \mathbf{R}_{\mathrm{NED}}^{\mathrm{PRCS}}\cdot\mathbf{R}_{\mathrm{BODY}}^{\mathrm{NED}}\cdot\left\{\mathbf{t}_{\mathrm{SOCS}}^{\mathrm{BODY}} + \mathbf{R}_{\mathrm{SOCS}}^{\mathrm{BODY}}\cdot\mathbf{R}_{\mathrm{AUCS}}^{\mathrm{SOCS}}\cdot\begin{pmatrix}0\\0\\r\end{pmatrix}\right\} \quad (5)$$

where $\mathbf{q}$ = trajectory point in PRCS
$\mathbf{R}_{\mathrm{NED}}^{\mathrm{PRCS}}$ = rotation matrix from NED to PRCS
$\mathbf{R}_{\mathrm{BODY}}^{\mathrm{NED}}$ = attitude rotation matrix (angles roll, pitch, yaw)
$\mathbf{t}_{\mathrm{SOCS}}^{\mathrm{BODY}}$ = lever arm offsets
$\mathbf{R}_{\mathrm{SOCS}}^{\mathrm{BODY}}$ = rotation matrix based on boresight angles
$\mathbf{R}_{\mathrm{AUCS}}^{\mathrm{SOCS}}$ = rotation matrix from AUCS to SOCS
r = laser range
$\mathbf{p}$ = point in PRCS

The rotation from AUCS to SOCS itself does not contain uncertainties. Additionally, the transformation from SOCS to the BODY coordinate system (i.e., scanner boresight and lever arm calibration) is assumed to be error-free. The uncertainty values σ_{roll}, σ_{pitch}, σ_{yaw} of the attitude angles (interpolated from the trajectory data at the point's timestamp) are taken into account when transforming from the BODY system to the North-East-Down (NED) system. Finally, when transforming from the NED system to the Project Coordinate System (PRCS), the position coordinate uncertainties of the trajectory $\sigma_{\mathrm{traj,N}}$, $\sigma_{\mathrm{traj,E}}$, $\sigma_{\mathrm{traj,D}}$ are considered to estimate the point's coordinate uncertainties with respect to the PRCS.

2.1.2 Bathymetric TPU Estimation Model:

In case of the bathymetric TPU estimation model (Figure 2), we use a plane describing the air-water interface ("water surface") in the vicinity of the laser ray's intersection point I, including uncertainty measures of this plane (all given with respect to PRCS). The latter two involve the z-uncertainty value ($\sigma_{z,\mathrm{WSM}}$) and the uncertainty value of the plane's normal vector ($\sigma_{\mathbf{n},\mathrm{WSM}}$). In fact, both uncertainty values result from interpolation of the grid-based water surface model at the intersection point's xy position.

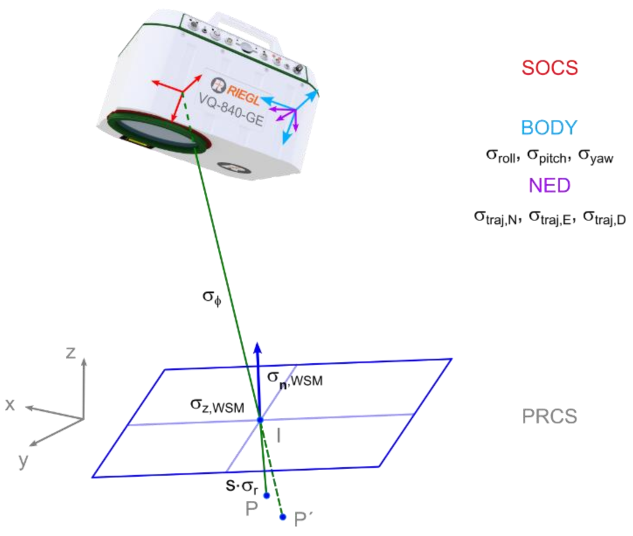

Figure 2. Ray-based model being used for TPU estimation in case of a bathymetric point.

First, the coordinate uncertainties of intersection point I with respect to PRCS are estimated. This can be done in a similar way as for the points in the topographic TPU estimation model. However, instead of using σ_r (which is relevant for the ray's subaqueous part in this case), we estimate the uncertainty $\sigma_{r_{\mathrm{air}}}$ of the aerial range part r_{air} instead:

$$\sigma_{r_{\mathrm{air}}} = \frac{n_z \cdot \sigma_{z,\mathrm{WSM}}}{\cos\theta_I} \tag{6}$$

where n_z= z-component of the water surface normal vector
$\sigma_{z,\mathrm{WSM}}$= z-uncertainty value of the WSM
θ_I= angle of incidence of the laser ray

For the uncertainty estimation of the intersection point I, Equations (4) and (5) are used, but r and σ_r are replaced with r_{air} and $\sigma_{r_{\mathrm{air}}}$.

The uncertainty value of the plane's normal vector $\sigma_{\mathbf{n},\mathrm{WSM}}$ takes effect when estimating the uncertainty values of the components of the subaqueous ray direction vector $\mathbf{u}_{\mathrm{water}}$ based on Snell's law:

$$\mathbf{u}_{\mathrm{water}} = \eta \cdot \mathbf{u}_{\mathrm{air}} + \left(\eta \cdot k - \sqrt{1 + \eta^2 \cdot (k^2 - 1)}\right) \cdot \mathbf{n} \tag{7}$$

where η = ratio of the refractive indices of air and water
$\mathbf{u}_{\mathrm{air}}$= aerial direction vector (i.e. to intersection point)
$\mathbf{n}$ = water surface normal vector
$k = -\mathbf{n}^T\mathbf{u}_{\mathrm{air}}$

Considering the laser range uncertainty σ_r (scaled by a factor s accounting for the lower group velocity of the laser in water compared to that in air), we finally estimate the coordinate uncertainties of the refraction-corrected point with respect to PRCS:

$$\mathbf{p} = \mathbf{i} + r_{\mathrm{water}} \cdot \mathbf{u}_{\mathrm{water}} \tag{8}$$

where $\mathbf{i}$ = intersection point in PRCS
r_{water}= subaqueous range part
$\mathbf{u}_{\mathrm{water}}$= subaqueous ray direction vector
$\mathbf{p}$ = point in PRCS

Note, that a well-defined water surface model is crucial for estimating the coordinate uncertainties. However, there may be areas with sparse or even missing water surface points. In those areas, the water surface model is interpolated from the neighborhood but the uncertainty values are not, i.e. they get an invalid value indicating that a reliable uncertainty estimation is not possible there. Due to this conservative approach, bathymetric points having their intersection point close to those areas also obtain invalid coordinate uncertainty values.

2.1.3 Comparison to Alternative Approaches:

As shown in the previous sections, we use a simple ray-based model for TPU estimation, i.e. without taking beam divergence into account.

The tool cBLUE allows to specify environmental parameters such as wind speed (by selecting one of 5 predefined wind speed categories) and turbidity (by selecting one of 5 predefined turbidity categories). Our approach uses a grid-based water surface model based on previously classified water surface points in order to model surface roughness but does not consider turbidity nor the interaction of the laser beam within the water column (e.g. scattering effects). On the other hand, the usage of the water surface model allows to handle inhomogeneous waviness conditions within a flight campaign or even within one flight strip (e.g. covering both areas within wind-protected bays and in rough open sea) as it often occurs in practice.

Additionally addressing laser beam propagation in the water column (depending on turbidity) would be a further improvement of the model. However, from a practical point of view, precise local environmental input data (primarily turbidity, but also water temperature and/or salinity) may not be available. This is for example the case, when no environmental service provider is covering the mission area, and no survey team is deployed on land or on boats simultaneously to the airborne data acquisition campaign. For large areas, local differences and potential change of conditions over the duration of the data acquisition would have to be considered in order to choose environmental parameter input with adequate precision. It can therefore be assumed that entering approximative values would not refine the calculation, but rather risk to distort the result.

2.2 Workflow

RIEGL TPU Estimator is available directly in RiHYDRO, the processing software suite dedicated for the post-processing of bathymetric LiDAR data which is an add-on of RiPROCESS (RiHYDRO, 2025). It is, however, not a mandatory step in data processing but serves solely as quality check on-demand.

After classification of the water surface points, uncertainty values for their coordinates are estimated (using the topographic TPU estimation model as described in the previous section). Based on those points and their uncertainties, the WSM is generated including the estimation of z-value uncertainty and surface normal vector's angular uncertainty for each grid position (Note, that uncertainty values may be valid or not – see end of section 2.1.2).

In the next processing step, the refraction correction process based on the WSM, the following information is provided for each laser ray:

1. intersection point of laser ray with water surface mode
2. surface normal at this point
3. WSM uncertainty values interpolated from neighboring WSM grid cells at this point (valid only if all those neighboring grid cells have valid uncertainty values)

Using this information and applying variance propagation to each refracted laser ray, the uncertainties (including THU and TVU) of the points below the WSM are finally estimated.

3. Examples

In the following we illustrate the application of the RIEGL TPU Estimator tool on bathymetric datasets from coastal surveys. The first example serves for a comparison with TVU values resulting using cBLUE.

In the second example we give an outlook how RIEGL TPU Estimator is prepared for quality control to check the resulting values against a specified threshold. A raster map enables identification of areas where the required criteria are not met and thus also serves as a basis for planning additional survey missions.

3.1 Example 1: Comparison with cBLUE

3.1.1 Project Description: The dataset used for the comparison was recorded using a VQ-860-G on August 24, 2024, at the Adriatic coast at Samer island south of Rovinj, Croatia. The flight altitude was around 300 m above ground level. Figure 3 presents an overview of the covered area and indicates the coastal features as well as the topography.

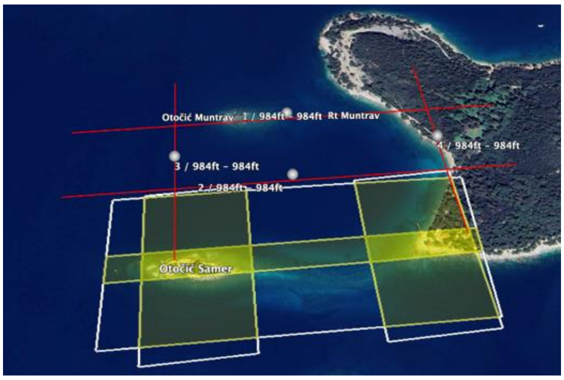

Figure 3. Geographic situation of the project area near Rovinj, Croatia (source: Google Earth). Area coverage and flight lines. Both the shallow water passage between the mainland and the island Samer, and the steeper coast to the south and west of the island were covered. The profiles shown in Figures 4, 5, and 6 are taken from the southern coast of the island.

3.1.2 Results: In cBLUE, *RIEGL* VQ-880-G was selected as the 'scanner model' for the calculation, since the more recent instrument type *RIEGL* VQ-860-G is not yet supported.

As both scanners use a similar scanning mechanism, we considered it legitimate to neglect the model type for the present comparison to highlight the generic differences of the two methods.

As water surface settings, a wind speed in the range of 0 to 2 knots (“calm-light air”) was selected because for the major part of the flight line, no significant surface waves were observed in the scan data. As turbidity, an attenuation coefficient in the range of 0.06 to 0.10 m-1 (“clear”) was chosen, due to the good depth penetration capability observed in the data.

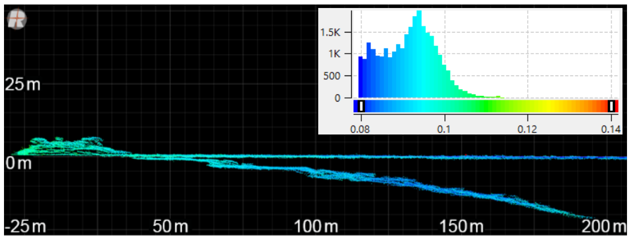

Figure 4. Estimated TVU values close to a shoreline (profile view) calculated with RIEGL TPU Estimator.

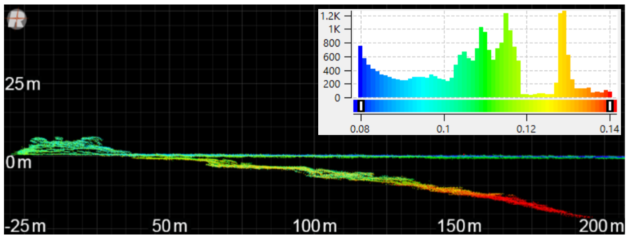

Figure 5. Estimated TVU values close to a shoreline (profile view) calculated with cBLUE.

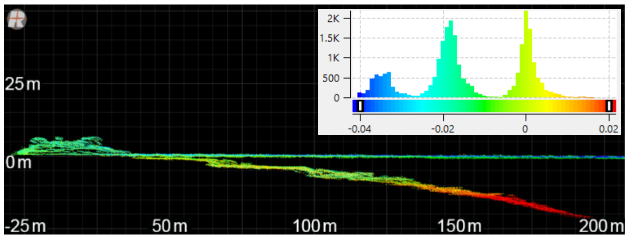

Figure 6. Differences in TVU values close to a shore line (profile view) cBLUE – RIEGL TPU Estimator.

When looking at Figures 4, 5, and 6, we observe similar absolute values for the TVU in the range of 8-12 cm but a significant depth-dependence of the cBLUE result which the RIEGL TPU Estimator does not confirm. An explanation for this may be found in the influence of the water surface model on the TVU. As described above, entirely different mechanisms are applied here, and the effect may be the observed difference. There is also a depth dependence in the RIEGL TPU Estimator’s results, however on a different scale than that of cBLUE and thus not visible in Figure 4. This is subject to further investigation.

Another observation unrelated to the comparison with cBLUE concerns the amplitude: The RIEGL TPU Estimator’s results feature an amplitude-dependency which is mainly caused by the uncertainty of the laser range σ_r which increases with decreasing amplitude. This amplitude dependency is an important instrument-specific property affecting TPU estimation. In this example, however, the influence is merely in the mm-range. Nevertheless, the rocky seafloor in this dataset provides sufficient amplitude variation to significantly affect the TVU values.

To further investigate the robustness of the method and assess the origins of the discrepancies shown here, a more comprehensive comparison applying both methods to other datasets is envisaged.

3.2 Example 2: Validation against IHO Standard

3.2.1 Project Description: The dataset used for exemplifying a quality control procedure using RIEGL TPU Estimator was recorded on February 18, 2025, at the Baltic coast near Rostock, Germany, using a *RIEGL* VQ-840-GE. The flight altitude was around 300 m above ground level. Figure 7 shows the project area and the mission flight lines.

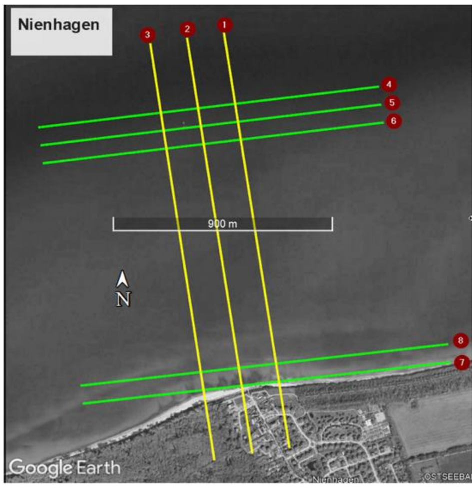

Figure 7. Geographic situation of the study area (source: Google earth) and mission flightlines at the beach of Nienhagen near Rostock, Germany, in the Baltic Sea.

3.2.2 Results: We chose a grid representation for direct visual detection of discrepancies between the results and defined threshold values. For this example, we decided to compare with values required in IHO Exclusive Order. The maximum allowed values for THU and TVU are taken from the tables given in IHO standard S-44 or are derived from applying the calculation formulas given therein. The vertical datum from the German Geoid Model DHHN 2016 (Deutsches Haupthöhennetz) is used as water depth reference.

In the creation of the grid, a cell is only assigned with a valid TPU value, when all points lying within the cell have a valid TPU value. The maximum value of all points within the cell is taken as representative cell value. The purple points on the edge of the strip shown in Figure 8 therefore indicate areas where at least one point per cell has an invalid value. The grids in Figure 8 and Figure 9 only consider the classified seafloor points from the forward-looking part of the scan segment. One could as well consider both scan segments or even more than one flight strip for raster model generation, depending on the defined specifications of the deliverable TPU report. We chose to use a single segment to avoid side effects making interpretation of the results more difficult.

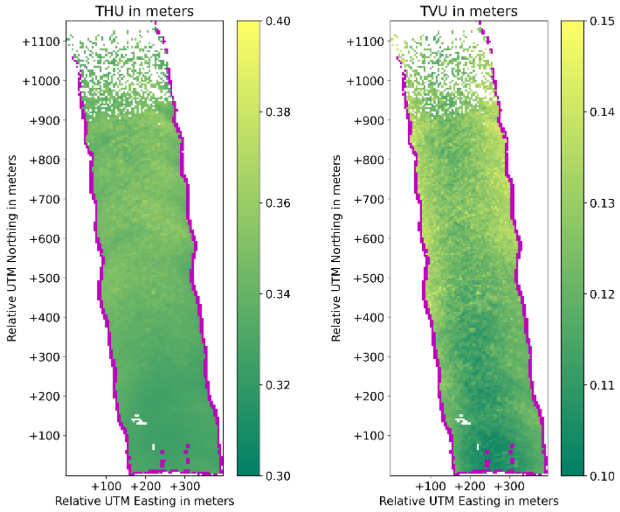

Figure 8. Estimated THU values (left) and TVU values (right) in a raster model with 5m resolution.

It can be observed, that the values for THU and TVU of sea ground points slightly increase with depth. Also, noise increases with higher roughness of the water surface. This tendency is consistent with expected results and highlights the necessity to observe recommended limitations in mission planning.

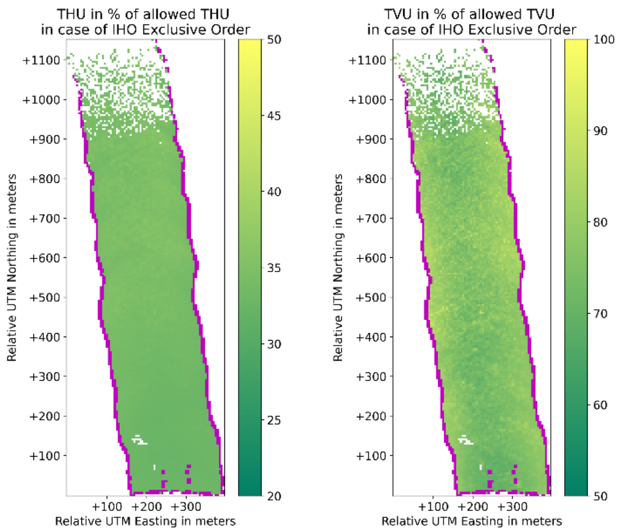

Figure 9. Estimated THU values relative to allowed THU values (left) and estimated TVU values relative to allowed TVU values (right) in a raster model with 5 m resolution.

As seen in Figure 9, valid relative THU values lie between 32% and 38%, hence, significantly below the allowable threshold of IHO Exclusive Order (which is 1 m). Valid relative TVU values lie between 64% and 91%, therefore also meeting the requirements, as resulting from applicable formula that takes into account depth dependency).

Another aspect that can be observed is the dependency of the TPU values on the laser incident angle.

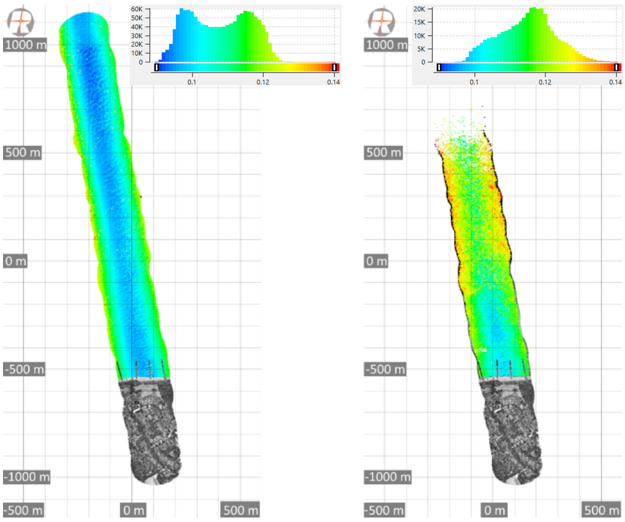

Figure 10. Scan angle dependency for TVU for water surface (left) and seafloor (right).

Figure 10 shows the change of TVU values in one scan strip depending on the point position: The values increase towards the strip edges, resulting from the scan angle as a consequence of the scan mechanism which provides off-nadir angles of ±20° at the edges of the swath and ±14° in the swath's center.

Figure 10 and Figure 11 each show the points from the water surface on the left side, and the points from the seafloor on the right side, plus reflectance-coded points from the onshore area. Both scan segments (forward and backward) are used for this illustration.

In summary, while the incidence angle effect due to the described scan mechanism is a fixed property observable in the results, the project-specific dominating impact on TVU/THU are the physical properties of the water surface at the time of data acquisition. The results may therefore be significantly different at different seastate and depending on the quality of the trajectory.

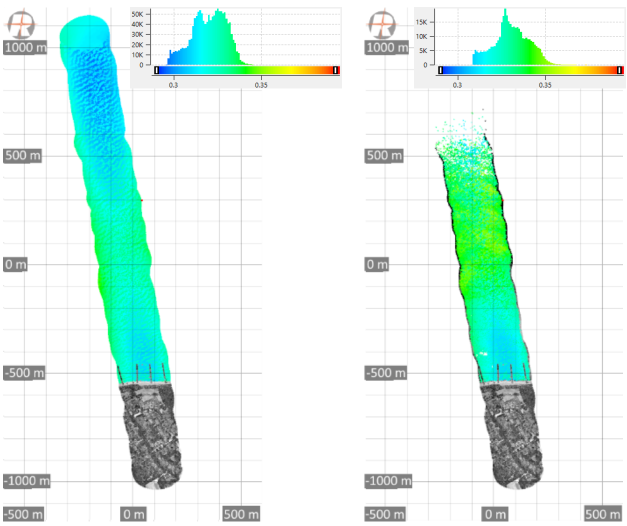

Figure 11. Scan angle dependency for THU for water surface (left) and seafloor (right).

4. Discussion and Outlook

An algorithm for calculating total propagated uncertainty estimations for Bathymetric LiDAR based on sensor properties and measurement results alone is presented. This stands in contrast to other models adding environmental parameters into the calculations. We discuss resulting TVU and THU from two measurement campaigns at different coastal areas in Europe with distinctly different topographic and hydrographic characteristics.

The first dataset features a rocky island in Croatia and is used to compare our results with those from applying cBLUE. This example serves to prove and validate the approach: In testing the proposed method, differences concerning the increase of TVU with depth could be observed in comparison with methods using physical properties of the water column as input. This matter will be addressed in follow-up investigations. It is planned to use ground-truth to validate or falsify our model assumptions.

The second dataset has been collected over a sandy beach in the Baltic Sea. For this example, we analyze how the TVU and THU results can be used to assess whether the dataset meets predefined criteria – in this case, IHO Exclusive Order. It therefore serves as an outlook on how LiDAR TPU estimation could be used in hydrographic practice for quality control without aggravation of the hydrographer's workload or expenses: The tool could be embedded in a user interface for evaluating datasets by attributing TPU values to tiled control charts. The graphical chart representation used in the example is intended to provide immediate visual comprehensibility of areas in which the thresholds are not reached. Other formats, such as tabular or statistical result displays, could be created to comply with formal requirements for deliverables if given in a hydrographic standard or another organization's quality norm.

A potential further use of TPU estimation tools might be an a-priori assessment in the course of mission planning: Analogous to how the technical performance specifications of a measurement system are used for a basic assessment of the achievability of required measurement results in terms of e.g. point density, depth penetration or coverage, TPU, pre-calculated on the basis of the parameters known up-front like flying altitude, coarse depth, and scanner properties could support an estimate whether certain TPU requirements will probably be met, and, if needed, the mission layout can be adapted accordingly. It is stressed, however, that this workflow is not supported by the current stage of development of the tool.

Finally, it should not be neglected to consider specific processing methods available for state-of-the art bathymetric LiDAR that are used in the surveying practice to improve mission efficiency, and the possible implications for TPU estimation when used for datasets on which these processing options have been applied. As such, we would like to mention the use of averaging methods for significant improvement of the depth readability of bathymetric datasets in post-processing (Schwarz et al., 2024). We would like to emphasize that the described algorithm currently implemented in the RIEGL TPU Estimator is not yet applicable on averaged data as certain aspects explicitly address single measurements. It is therefore planned to add TPU features also for averaged data in a next step.

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