# Comparing spatial patterns in raster data using R

Jakub Nowosad[1]

[1]Institute of Geoecology and Geoinformation, Adam Mickiewicz University, Poznań, Poland - nowosad.jakub@gmail.com

## Abstract

Spatial patterns are a crucial aspect of geographical studies and play a significant role in various fields, such as remote sensing, ecology, and geology. Comparing spatial patterns is a common task in these fields, and using raster data is a popular approach. This work aims to provide a comprehensive overview of existing R packages for comparing spatial patterns in rasters. First, we discuss various methods for comparing spatial patterns, including visual inspection, correlation coefficient between focal regions of two rasters, spatial autocorrelation analysis, structural similarity index, and comparison of spatial signatures. These methods can be applied to continuous or categorical raster data, and the choice of method depends on the type of data and the specific research question. Next, we present several R packages that implement these methods, including terra, SSIMmap, and spatialEco, and show how they can be applied. These packages provide a wide range of tools for comparing spatial patterns in rasters, offering researchers a powerful means to study change, similarity, association, and model assessment. However, we also highlight issues with the current software, such as the absence of standardization and inadequate documentation. Importantly, there is still a lack of studies that systematically compare different methods and suggest good practices in their use. The growing number of FOSS tools for spatial raster comparison offers a promising avenue for testing various methods and their application to real-life scenarios. Further research is needed to evaluate the performance of these methods and establish best practices for their use.

## 1. Introduction

Dale (2000) defines a spatial pattern as a scale-dependent predictability of the physical arrangement of observations. Spatial patterns are a crucial aspect of many geographical studies. They are an inherent property of spatial data and can take various forms, depending on the type of data and the underlying processes. Here, we focus on spatial patterns in rasters, though the same principles can be relevant for other spatial data types, such as vector data and point clouds.

Comparison of spatial patterns is a common task in many fields, including remote sensing, ecology, and geology. It can be used to study change, to study similarity, to study association, and for spatial model assessment (Long and Robertson, 2018). It may also encapsulate many types of operations: comparing the same variable(s) for different areas, comparing different datasets (e.g., different sensors), or comparing the same area but at different times.

To answer various needs, many methods have been developed to compare spatial patterns, both for continuous and categorical raster data, at different scales of comparison, and for different data dimensions. This work aims to provide an overview of existing R packages for comparing spatial patterns in rasters. We start by discussing the methods for comparing spatial patterns and then present R packages that implement these methods. Finally, we discuss what is currently missing and what could be improved.

## 2. Methods for comparing spatial patterns

The primary method for comparing spatial patterns is a visual inspection. It takes advantage of the human ability to recognize patterns and considers many aspects, including local and global similarities, and logical coherence (Hagen-Zanker, 2008). However, both the observer's perspective and the chosen visualization method can heavily influence the results, making it a subjective process.

This section presents computational methods for comparing spatial patterns between two raster layers. We also briefly discuss the possibility of comparing two sets of continuous raster layers (such as time series) or many layers of categorical raster data. For more complete reviews of existing methods, we suggest reading Kuhnert et al. (2005), Foody (2007), and Long and Robertson (2018).

Representation of the patterns in spatial raster data depend on the type of data. We may think of spatial patterns for continuous rasters (e.g., elevation) as an interplay between intensity and spatial autocorrelation or between composition and configuration for categorical rasters (e.g., land cover) (Gustafson, 1998). Intensity relates to the range and distribution of values of a given variable, while spatial autocorrelation is a tendency for nearby values of a given variable to be more similar than those that are further apart. On the other hand, composition is the number of cells belonging to each discrete category, while configuration represents their spatial arrangement. Thus, the type of data determines the methods that can be used to compare spatial patterns. To highlight the differences between the two types of data, Table 1 provides examples of methods for comparing two layers of spatial continuous raster data, while Table 2 gives examples of methods for comparing two layers of spatial categorical raster data.

There are several additional considerations besides data type when comparing spatial patterns using computational methods. Here, we focus on three general dimensions relevant to comparing raster data. They include the type of outcome (raster, single value, multiple values), the context of the comparison (non-spatial, spatial), and the possibility of comparing disjoint regions.

The raster outcome is the most straightforward type, producing

| Examples | Outcome | Context | Disjoint areas |
|---|---|---|---|
| The difference between values of two rasters for each cell | raster | non-spatial | no |
| Correlation coefficient between focal regions of two rasters; The difference between a focal measure of two rasters (e.g., based on a GLCM-based texture measure (Haralick et al., 1973)); Spatial autocorrelation analysis of the differences (Cliff, 1970); Structural Similarity Index (Robertson et al., 2014; Wang et al., 2004) | raster | spatial | no |
| Statistics of the differences between rasters' values (e.g., RMSE) | single value | non-spatial | no |
| Dissimilarity between the distributions of two rasters' values | single value | non-spatial | yes |
| Average of Structural Similarity Index (Robertson et al., 2014; Wang et al., 2004); Complex Wavelet Structural Similarity (Malik and Robertson, 2020; Sampat et al., 2009); Comparison of deep learning-based feature maps using a dissimilarity measure (Malik and Robertson, 2021) | single value | spatial | no |
| Comparison of deep learning-based feature maps using a dissimilarity measure (Malik and Robertson, 2021) | single value | spatial | yes |
| The distribution of the difference between values of two rasters; Statistics of the differences between rasters' values (e.g., RMSE) calculated at many scales | multiple values | non-spatial | no |

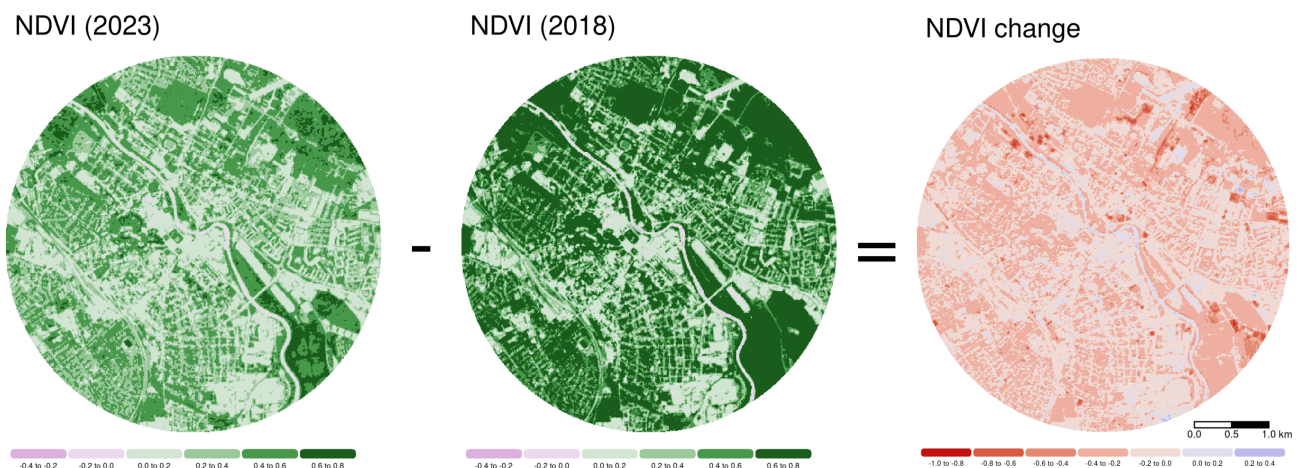Table 1. Methods for comparing two layers of spatial continuous raster data.



Figure 1. An example of rasters' comparison resulting in a raster outcome: the difference between two rasters.

a new raster with the same dimensions as the input rasters (Figure 1). Methods resulting in a raster outcome are usually limited to comparing two rasters with the same dimensions (spatial resolution and extent), and they often rely on basic approaches such as creating a difference map or a binary map of differences. These strategies facilitate the visualization of local differences between the two rasters, enabling further analysis and interpretation of the observed discrepancies. However, the cell-by-cell approaches may not be sufficient, as they do not take into account the spatial context of the data, e.g., small drifts of values in a raster (Pontius, 2002). More complex methods, such as calculating the correlation coefficient between focal regions of two rasters, can be used to address this issue, as they incorporate relations of neighboring cells. Such methods are sometimes referred to as "pattern based map comparison techniques" (White, 2006). Their disadvantage is that they lose local spatial information and are computationally expensive.

Next, the single value outcome is a numerical representation that condenses the differences between two rasters into one value (Figure 2). This approach is particularly suitable when

the goal is to apply the comparison to many regions (e.g., many pairs of rasters or two rasters divided into many zones) as it allows many types of further analysis (e.g., regression, clustering, segmentation).

Many single value methods just summarize the differences between two rasters' values (*non-spatial*), such as the proportion of changed pixels or the overall comparison. Methods that result in a single value outcome can consider the spatial context of the data (*spatial*). Some methods, such as the Structural Similarity Index, fuzzy Kappa index, or V-measure, are especially suited to comparing spatial patterns by taking into account the spatial relations between values in the rasters.

Additionally, some of the methods can be used to compare both non-disjoint and disjoint areas. Hagen-Zanker (2008) distinguishes between methods that are able to compare overlapping areas and those that are not. The former (*non-disjoint*) are often used to compare the same area at different times or for other data products, while the latter (*disjoint*) are used to either compare the same area or compare different regions. Comparing disjoint
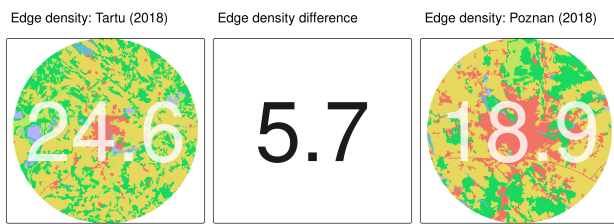
Figure 2. An example of rasters' comparison resulting in a single value outcome: the difference between edge densities of two rasters.

areas is often done to determine whether the observed patterns were generated by the same process (Csillag and Boots, 2005).

Both, the methods with a raster outcome and a single value outcome include approaches inspired by content-based image retrieval (Kato, 1992) that use spatial signatures to represent spatial patterns and dissimilarity measures to compare them (Jasiewicz and Stepinski, 2013). A spatial signature is any numerical representation (compression) of a spatial pattern. For a continuous raster, it can be one or more texture measures based on gray-level co-occurrence matrix (GLCM) or a deep learning-based feature map, while for a categorical raster, it can be a set of landscape metrics or a co-occurrence vector of classes. Then, having spatial signatures for both areas (or sensors/ moments), we can compare them using a dissimilarity measure (e.g., Euclidean distance, cosine similarity, etc.) (Cha, 2007).

The outcome can also consist of multiple values (Figure 3). Examples of such methods include the confusion matrix, which is often used to compare the results of a classification, or the visualization of the distribution of the difference between values of two rasters. They are often used to summarize the results of the comparison.

Many methods presented in Table 1 and Table 2 can be further extended to compare two sets of raster layers, such as two time-series. The most basic approach in such cases is to treat each layer as a separate raster and compare corresponding layers using the methods described above (a pairwise comparison). The outcomes of such comparisons are many rasters (instead of one), sets of single values, or groups of multiple values that can be further analyzed. With these results, we can, for example, derive the average difference raster or the average RMSE between two time-series of rasters for each cell. Another approach is to compress the data into one dimension (e.g., one raster layer) and then compare such compressed data. Finally, having two sets of rasters opens other possibilities, such as creating temporal or spatiotemporal signatures and comparing them using dissimilarity measures. For instance, we can extract values of each cell for every layer or calculate the average value of a focal region for each cell and then compare these signatures using a dissimilarity measure.

## 3. R packages for comparing spatial patterns

R (R Core Team, 2024) is a popular programming language for working with spatial data, including in remote sensing, ecology, and soil science. It provides various packages for reading, processing, analyzing, and visualizing spatial raster data. This also includes packages for comparing spatial patterns between two rasters, which we present in this section. However, it is worth

mentioning that methods implemented in R packages are not limited to the categories presented above, with some packages implementing multiple methods or providing more than one type of comparison.

The most fundamental packages for working with raster data in R are **terra** (Hijmans, 2024) and **stars** (Pebesma and Bivand, 2023). Both have their own classes for representing raster data and provide a wide range of general spatial operations, including raster algebra. Thus, they can be directly used to compare spatial patterns in raster data using the cell-by-cell methods, such as the difference between values of two rasters for each cell and the binary difference between two rasters (with raster outcome), the proportion of changed pixels (with single value outcome), or the distribution of the difference between values of two rasters and confusion matrix (with multiple values outcome). Comparing the values in each cell is also a basis for calculating various statistics, e.g., ones often used in validating machine learning models, such as RMSE. Many R packages provide sets of such metrics, including **Metrics** (Hamner and Frasco, 2018), **metrica** (Correndo et al., 2022), and **yardstick** (Kuhn et al., 2024). Multiple metrics at many spatial scales can be calculated with `ww_multi_scale()` from the **waywiser** package (Mahoney, 2023).

Additionally, the **terra** package has a few other tools that can be helpful for spatial pattern comparison. It includes the dedicated function `focalPairs()` for calculating the correlation coefficient between focal regions of two rasters and `autocor()` to compare the spatial autocorrelation of the differences between two rasters. That package also gives tools to apply various moving window operations, including `focal()` and `focal3D()`, which can be used to calculate a selected focal measure of two rasters.

The categorical rasters can be compared using **spatialEco** (Evans and Murphy, 2023) that provides a function `raster.change()` with various statistics calculated in a moving window, including two-tailed paired t-test and cross-entropy loss function. The **SSIMmap** package (Ha and Long, 2023) compares spatial patterns of two continuous rasters using the Structural Similarity Index (SSIM) for each cell and a whole area. SSIM, along with the similarity index (CQ) based on the co-dispersion coefficient, modified t-test of spatial association, and Tjostheim's coefficient, is also available in the **SpatialPack** package (Vallejos et al., 2020). The **rasterdiv** computes various diversity indices based on information theory (Rocchini et al., 2021) in a moving window that are applicable to both continuous and categorical rasters. Next, **diffeR** (Pontius Jr. and Santacruz, 2023) offers a set of metrics to compare continuous and categorical rasters at original and multiple resolutions, such as the overall difference and mean absolute deviation (MAD). The spatial patterns of two categorical rasters can be compared using the **sabre** package (Nowosad and Stepinski, 2018), which returns two maps and three values: V-measure, homogeneity, and completeness.

One of R's strengths is the availability of many specialized packages for computing metrics to characterize spatial patterns. The general tool to calculate such metrics for categorical rasters is the **landscapemetrics** package (Hesselbarth et al., 2019) that offers more than 130 metrics to describe the composition and configuration of spatial patterns, while various metrics for continuous rasters can be calculated using the **geodiv** package (Smith et al., 2023). Moreover, **GLCMTextures** (Ilich, 2020) calculates GLCM texture metrics based on the spatial arrangement of pixel values in a raster. All three packages allow the calculation of

| Examples | Outcome | Context | Disjoint areas |
|---|---|---|---|
| The binary difference between two rasters | raster | non-spatial | no |
| The difference between a focal measure of two rasters (e.g., selected landscape metric); Dissimilarity between spatial signatures of focal regions of two rasters | raster | spatial | no |
| The proportion of changed pixels; The overall comparison (Pontius, 2002), A statistic of the differences between rasters' values | single value | non-spatial | no |
| Comparison of the values of a non-spatial landscape metric | single value | non-spatial | yes |
| Multiple resolution procedure (Costanza, 1989); Expanding window approach (Kuhnert et al., 2005); Fuzzy Kappa (Hagen-Zanker, 2009); Spatial association between regionalizations using V-measure (Nowosad and Stepinski, 2018) | single value | spatial | no |
| Comparison of the values of a landscape metric (Turner et al., 1989) or fractal dimensions (Batty and Longley, 1994); Dissimilarity of a spatial signature between two rasters (Jasiewicz and Stepinski, 2013) | single value | spatial | yes |
| The confusion matrix | multiple values | non-spatial | no |
| Comparison of mutual information spectra (Remmel and Csillag, 2006) | multiple values | spatial | no |

Table 2. Methods for comparing two layers of spatial categorical raster data.

the metrics for the whole raster or for focal regions of the raster. Thus, they can be used to calculate a selected metric for each cell in two rasters, with the results as either a difference raster or a single value. The single value outcome is also possible with the **bespatial** package (Nowosad, 2024a) that calculates several entropy metrics for spatial data inspired by Boltzmann's entropy formula and **fractaldim** that calculates fractal dimensions of spatial patterns (Sevcikova et al., 2021).

An alternative approach is to calculate several metrics simultaneously for each raster (*spatial signatures*) and then compare them using a dissimilarity measure. This needs to be done carefully, as the choice of metrics, how they are normalized and weighted, and the selection of the dissimilarity measure can significantly affect the results (Niesterowicz and Stepinski, 2006). For categorical rasters, spatial signatures incorporating both composition and configuration, such as co-occurrence vector (*cove*), can be calculated using the **motif** package (Nowosad, 2021). This package also allows the comparison of such signatures using one of many dissimilarity measures. A more general tool to compare spatial signatures for two rasters is a `spq_compare()` function from **spquery** (Nowosad, 2023) that expects two multilayer rasters and a dissimilarity measure. Each layer in those rasters is treated as a part of the spatial signature for the given cell, and the dissimilarity measure is calculated independently for each cell. Spatial signatures could also be based on deep learning-based feature maps created with packages such as **keras3** (Kalinowski et al., 2024) and **torch** (Falbel and Luraschi, 2023). R packages that provide various dissimilarity measures useful for the abovementioned comparisons include **proxy** (Meyer and Buchta, 2022), **philentropy** (Drost, 2018), and **emdist** (Urbanek and Rubner, 2023).

## 4. Code examples

To show how to apply the presented R packages for comparing spatial patterns, we provide two sets of R code examples at https://github.com/Nowosad/comparing-spatial-patterns-2024 (Nowosad, 2024b). The first one is based on continuous raster data representing normalized difference vegetation index (NDVI) values derived from Sentinel-2 images for early summer

of 2018 and 2023 in the area of Tartu, Estonia and the area of Poznan, Poland. The second one uses categorical raster data of CORINE Land Cover for Tartu and Poznan from the years 2000 and 2020. The source data for these examples come from the Copernicus Programme (European Space Agency, 2022) and the Copernicus Land Monitoring Service (European Environment Agency, 2019), respectively.
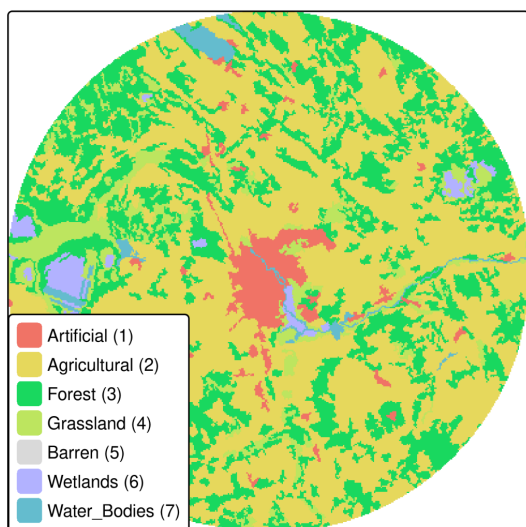
## 5. Discussion

This work provides an overview of the methods for comparing spatial patterns and presents R packages that implement these methods. It also gives practical examples of how to use these packages for two types of spatial raster data: continuous and categorical.

R and its packages provide a wide range of tools for comparing spatial patterns. These tools can be used to compare spatial patterns in raster data, both continuous and categorical, at different scales and dimensions. At the same time, R implementations of some commonly used methods are still missing, such as the Complex Wavelet Structural Similarity Index (Malik and Robertson, 2020; Sampat et al., 2009) or the Fuzzy Kappa index (Hagen-Zanker, 2009). Not all R packages are also available on CRAN, which may limit their discovery. Several of the presented packages have minimal documentation and examples, which may hinder their use by the broader community. The efforts needed to develop user tutorials and maintain scientific software are still underappreciated in the scientific community and undervalued in the academic reward system. Merow et al. (2023) proposed several ways to improve the situation, including the creation of a new class of article for software updates, but the implementation of these suggestions is still limited.

Moreover, the current R efforts for comparing spatial patterns are rather fragmented, with packages having different interfaces, assumptions, and types of data they can handle. Consequently, users must invest significant time in learning how to use each tool, and the developers face the challenge of re-implementing similar functionality. While the diversity of packages brings numerous benefits, the lack of coordination and standardization presents downsides to working independently.

## Tartu (2000)



Legend:
- Artificial (1)
- Agricultural (2)
- Forest (3)
- Grassland (4)
- Barren (5)
- Wetlands (6)
- Water_Bodies (7)

### CLC 2018

| | **1** | **2** | **3** | **4** | **6** | **7** |
|---|---|---|---|---|---|---|
| **1** | 4821 | 357 | 2 | 10 | 0 | 0 |
| **2** | 1342 | 67915 | 684 | 389 | 0 | 0 |
| **3** | 59 | 415 | 33670 | 2896 | 9 | 22 |
| **4** | 122 | 638 | 1435 | 7040 | 229 | 24 |
| **6** | 0 | 3 | 13 | 20 | 2177 | 37 |
| **7** | 0 | 0 | 0 | 0 | 3 | 1995 |

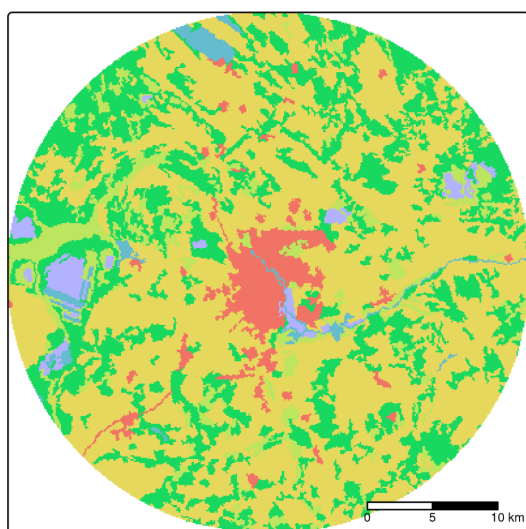(row labels: CLC 2000)

## Tartu (2018)



Figure 3. An example of rasters' comparison resulting in multiple values outcome: the confusion matrix.

Given the multitude of scientific questions and the almost unlimited range of possible map comparison methods (Hagen-Zanker, 2008), there is no universal method for assessing the similarity between spatial patterns. Thus, it is worth noting that the presented methods are not exhaustive, and there are many unresolved questions and things to consider. Boots and Csillag (2006) identified five key groups of issues that should be considered in undertaking spatial pattern comparison for categorical rasters, but which are also relevant for continuous rasters. These include which pattern characteristics is to be examined, which categories/ranges of values are involved in the comparison, selection of geographic space (entire region or subregions), choice on the unit of computed measures (e.g., pixel or moving window), and decision if the significance tests are needed. Other considerations include data preprocessing and the scale and extent of comparison (Tewkesbury et al., 2015). Moreover, different methods can produce varying outcomes when analyzing differences in spatial patterns, and using a variety of methods may be advisable to gain a comprehensive understanding. Thus, a careful selection of the methods and the tools is crucial to ensure the reliability and interpretability of the results.

There is, however, still a lack of studies that systematically compare different methods of assessing similarity between spatial patterns, or suggest good practices in their use. This deficiency is further compounded by the lack of code availability in many research papers, which limits not only the reproducibility of the results, but also the possibility of comparing different methods. Nevertheless, the growing number of FOSS tools for spatial raster comparison offers a promising avenue for testing various methods and their application to real-life scenarios.

### Acknowledgements

### References

Batty, Michael, and Paul Longley. 1994. *Fractal Cities: A Geometry of Form and Function*. San Diego, CA; London: Academic Press.

Boots, Barry, and Ferko Csillag. 2006. "Categorical Maps, Comparisons, and Confidence." *Journal of Geographical Systems* 8 (2): 109–18. https://doi.org/10.1007/s10109-006-0018-9.

Cha, Sung-Hyuk. 2007. "Comprehensive Survey on Distance / Similarity Measures Between Probability Density Functions." *International Journal of Mathematical Models and Methods in Applied Sciences* 1 (4): 300–307.

Cliff, Andrew D. 1970. "Computing the Spatial Correspondence Between Geographical Patterns." *Transactions of the Institute of British Geographers*, no. 50 (July): 143. https://doi.org/10.2307/621351.

Costanza, Robert. 1989. "Model Goodness of Fit: A Multiple Resolution Procedure." *Ecological Modelling* 47: 199–215.

Drost, HG. 2018. "Philentropy: Information Theory and Distance Quantification with R." *Journal of Open Source Software* 3 (26): 765.

European Environment Agency. 2019. "CORINE Land Cover 2018 (Raster 100 m), Europe, 6-Yearly - Version 2020_20u1, May 2020." European Environment Agency. https://doi.org/10.2909/960998C1-1870-4E82-8051-6485205EBBAC.

European Space Agency. 2022. "Sentinel-2 MSI Level-2A BOA Reflectance." https://doi.org/10.5270/s2_-znk9xsj.

Evans, Jeffrey S., and Melanie A. Murphy. 2023. *spatialEco*. Manual.

Falbel, Daniel, and Javier Luraschi. 2023. *Torch: Tensors and Neural Networks with 'GPU' Acceleration*. Manual.

Foody, Giles M. 2007. "Map Comparison in GIS." *Progress in Physical Geography: Earth and Environment* 31 (4): 439–45. https://doi.org/10.1177/0309133307081294.

Gustafson, Eric J. 1998. "Quantifying Landscape Spatial Pattern: What Is the State of the Art?" *Ecosystems* 1: 143–56. https://doi.org/dj6v74.

Ha, Hui Jeong (Hailyee), and Jed Long. 2023. *SSIMmap: The Structural Similarity Index Measure for Maps*. Manual.

Hagen-Zanker, A. 2008. "Measuring the Performance of Geo-simulation Models by Map Comparison." PhD thesis, Maastricht University. https://doi.org/10.26481/dis.20081210ah.

Hagen-Zanker, Alex. 2009. "An Improved Fuzzy Kappa Statistic That Accounts for Spatial Autocorrelation." *International Journal of Geographical Information Science* 23 (1): 61–73. https://doi.org/10.1080/13658810802570317.

Hamner, Ben, and Michael Frasco. 2018. *Metrics: Evaluation Metrics for Machine Learning*. Manual.

Haralick, Robert M, Karthikeyan Shanmugam, and Its' Hak Dinstein. 1973. "Textural Features for Image Classification." *IEEE Transactions on Systems, Man, and Cybernetics*, no. 6: 610–21.

Hesselbarth, Maximilian H. K., Marco Sciaini, Kimberly A. With, Kerstin Wiegand, and Jakub Nowosad. 2019. "*Landscapemetrics* : An Open-Source *R* Tool to Calculate Landscape Metrics." *Ecography* 42 (10): 1648–57. https://doi.org/gf4n9j.

Hijmans, Robert J. 2024. *Terra: Spatial Data Analysis*. Manual.

Ilich, Alexander R. 2020. "GLCMTextures." https://doi.org/10.5281/zenodo.4310186.

Jasiewicz, Jaroslaw, and Tomasz F. Stepinski. 2013. "Example-Based Retrieval of Alike Land-Cover Scenes From NLCD2006 Database." *IEEE Geoscience and Remote Sensing Letters* 10 (1): 155–59. https://doi.org/gjnqqx.

Kalinowski, Tomasz, JJ Allaire, and François Chollet. 2024. *Keras3: R Interface to 'Keras'*. Manual.

Kato, Toshikazu. 1992. "Database Architecture for Content-Based Image Retrieval." In *Image Storage and Retrieval Systems*, 1662:112–23. SPIE.

Kuhn, Max, Davis Vaughan, and Emil Hvitfeldt. 2024. *Yardstick: Tidy Characterizations of Model Performance*. Manual.

Kuhnert, Matthias, Alexey Voinov, and Ralf Seppelt. 2005. "Comparing Raster Map Comparison Algorithms for Spatial Modeling and Analysis." *Photogrammetric Engineering & Remote Sensing* 71 (8): 975–84. https://doi.org/10.14358/PERS.71.8.975.

Long, Jed, and Colin Robertson. 2018. "Comparing Spatial Patterns." *Geography Compass* 12 (2): e12356. https://doi.org/10.1111/gec3.12356.

Mahoney, Michael J. 2023. "Waywiser: Ergonomic Methods for Assessing Spatial Models." https://doi.org/10.48550/arXiv.2303.11312.

Malik, Karim, and Colin Robertson. 2020. "Exploring the Use of Computer Vision Metrics for Spatial Pattern Comparison." *Geographical Analysis* 52 (4): 617–41.

Malik, Karim, and Colin Robertson. 2021. "Landscape Similar-ity Analysis Using Texture Encoded Deep-Learning Features on Unclassified Remote Sensing Imagery." *Remote Sensing* 13 (3): 492. https://doi.org/10.3390/rs13030492.

Meyer, David, and Christian Buchta. 2022. *Proxy: Distance and Similarity Measures*. Manual.

Niesterowicz, Jacek, and Tomasz F. Stepinski. 2006. "On Using Landscape Metrics for Landscape Similarity Search." *Ecological Indicators* 64: 20–30. https://doi.org/f8cz4x.

Nowosad, Jakub. 2021. "Motif: An Open-Source R Tool for Pattern-Based Spatial Analysis." *Landscape Ecology* 36: 26–43. https://doi.org/10.1007/s10980-020-01135-0.

Nowosad, Jakub. 2023. *Spquery: Queries on Spatial Raster Data*. Manual.

Nowosad, Jakub. 2024. *Bespatial: Boltzmann Entropy for Spatial Data*. Manual.

Nowosad, J., and T. F. Stepinski. 2018. "Spatial Association Between Regionalizations Using the Information-Theoretical *V* -Measure." *International Journal of Geographical Information Science* 32 (12): 2386–2401. https://doi.org/gf283f.

Pebesma, Edzer, and Roger Bivand. 2023. *Spatial Data Science: With Applications in R*. London: Chapman and Hall/CRC. https://doi.org/10.1201/9780429459016.

Pontius Jr., Robert Gilmore, and Ali Santacruz. 2023. *diffeR: Metrics of Difference for Comparing Pairs of Maps or Pairs of Variables*. Manual.

Pontius, R Gil. 2002. "Statistical Methods to Partition Effects of Quantity and Location During Comparison of Categorical Maps at Multiple Resolutions." *Photogrammetric Engineering*.

R Core Team. 2024. *R: A Language and Environment for Statistical Computing*. Manual. Vienna, Austria: R Foundation for Statistical Computing.

Robertson, Colin, Jed A. Long, Farouk S. Nathoo, Trisalyn A. Nelson, and Cameron C. F. Plouffe. 2014. "Assessing Quality of Spatial Models Using the Structural Similarity Index and Posterior Predictive Checks." *Geographical Analysis* 46 (1): 53–74. https://doi.org/10.1111/gean.12028.

Rocchini, Duccio, Elisa Thouverai, Matteo Marcantonio, Martina Iannacito, Daniele Da Re, Michele Torresani, Giovanni Bacaro, et al. 2021. "Rasterdiv - An Information Theory Tailored R Package for Measuring Ecosystem Heterogeneity from Space: To the Origin and Back." *Methods in Ecology and Evolution* 12 (6): 2195. https://doi.org/10.1111/2041-210X.13583.

Sampat, M. P., Zhou Wang, S. Gupta, A. C. Bovik, and M. K. Markey. 2009. "Complex Wavelet Structural Similarity: A New Image Similarity Index." *IEEE Transactions on Image Processing* 18 (11): 2385–2401. https://doi.org/10.1109/TIP.2009.2025923.

Sevcikova, Hana, Don Percival, and Tilmann Gneiting. 2021. *Fractaldim: Estimation of Fractal Dimensions*. Manual.

Smith, Annie C., Phoebe Zarnetske, Kyla Dahlin, Adam Wilson, and Andrew Latimer. 2023. *Geodiv: Methods for Calculating Gradient Surface Metrics*. Manual.

Tewkesbury, Andrew P., Alexis J. Comber, Nicholas J. Tate, Alistair Lamb, and Peter F. Fisher. 2015. "A Critical Synthesis of Remotely Sensed Optical Image Change Detection Techniques." *Remote Sensing of Environment* 160 (April): 1–14. https://doi.org/10.1016/j.rse.2015.01.006.

Turner, Monica G., Robert Costanza, and Fred H. Sklar. 1989. "Methods to Evaluate the Performance of Spatial Simulation Models." *Ecological Modelling* 48 (1-2): 1–18. https://doi.org/10.1016/0304-3800(89)90057-4.

Urbanek, Simon, and Yossi Rubner. 2023. *Emdist: Earth Mover's Distance*. Manual.

Vallejos, Ronny, Felipe Osorio, and Moreno Bevilacqua. 2020. *Spatial Relationships Between Two Georeferenced Variables: With Applications in R*. Springer Nature.

Wang, Z., A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli. 2004. "Image Quality Assessment: From Error Visibility to Structural Similarity." *IEEE Transactions on Image Processing* 13 (4): 600–612. https://doi.org/10.1109/TIP.2003.819861.