A framework for risk assessment during natural hazard based on the Choquet integral and MACBETH

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Abstract

Risk assessment in the face of natural disasters is an increasingly critical issue with climate change. Evaluating risk involves two distinct assessments: first, the probability of the disaster occurring, and second, the potential stakes or impacts of such an event. These impacts are generally categorized along several dimensions, notably human factors and infrastructure concerns. However, even when focusing solely on human stakes, characterizing risk –particularly in terms of the potential to prevent casualties– is complex. The most common approach to this problem relies on a weighted sum, which assigns specific weights to each criterion. This method effectively establishes the relative importance of one criterion over another and has the advantage of being linear. However, it also presents certain limitations. In particular, its linearity prevents it from capturing interactions between criteria, such as veto effects. In this paper, we propose a risk mapping method that utilizes an alternative aggregation function. Specifically, we rely on the Choquet integral, which allows for the modeling of interactions between criteria, whether in terms of complementarity or substitutability, while maintaining desirable properties such as monotonicity.We evaluate our approach by applying it to the challenge addressed in the MOGEC project: coastal flooding projections under IPCC's RCP 8.5 scenario in the Pays-de-la-Loire region, following a storm of similar magnitude to Xynthia. For this assessment, we rely on the expertise of emergency response teams, for whom risk mapping is a key concern. Specifically, we demonstrate the advantages and limitations of our approach in the municipality of Batz-sur-Mer within this projected scenario.

Keywords: Risk assessment, natural hazard, submersion, decision making, Choquet integral, MACBETH

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1. Introduction

Mapping the risk is a major issue for first responders and decision makers. For first responders, it allows to determine which are the places they are willing to visit first when they need to respond to a natural disaster. For decision makers, it helps determining what are the changes they can make to urban planning in order to reduce the exposition of their territory to natural hazards.

Assessing risks presents several challenges. First, it is necessary to find ways to objectively analyze the situation through measurable indicators (*e.g.* ease to access the victims for first responders, presence of a safe zone or not, vulnerability of the population). This includes not only defining these indicators but also evaluating their consequences. One possible approach would be to create separate maps for each dimension of risk. However, this would result in a large number of maps, making interpretation difficult. Decision-makers would then need to synthesize these maps to identify the most at-risk areas and prioritize interventions accordingly.

One of the most classical ways to deal with risk mapping is to rely on the characteristics of the territory (altitude, buildings, roads. . .) and to assign a coefficient to each of them, representing the criticality of the characteristic. The map of the territory is transformed into a grid, with each of the cells being assigned a value for each of the characteristics. The risk is then computed through a weighted sum of a score on the characteristics, and assigned to each of the cells.

This approach presents some limitations. Some situation cannot be represented by weighted sums, as it is the case for veto situations –situations in which a low score in one of the criteria causes an overall low score, but high score does not necessarily implies an overall high score– but also more generally in cases of interaction criteria, whether they are complementary –fulfilling one criterion without another one is helpless– or substitutability –cases where any of a set of criteria being fulfilled is enough to get an overall good score. In order to overcome these limitations, it is possible to look at different aggregation functions.

A whole domain has focused on how to represent the preferences of a human decision maker, in particular when there are multiple criteria to take into account. This domain is Multiple Criteria Decision Aid (MCDA). In this article, we propose to rely on a MCDA approach to map the risks on a territory. More specifically, we rely on 2-additive Choquet integral, an approach that has been broadly and successfully applied to numerous situations (Grabisch and Labreuche, 2010). Determining the parameters –in our case the capacities (see definition 1)– of the model can be complicated for the decision maker. In order to help them provide their preference, we rely on an extension of the MACBETH method (Bana e Costa et al., 2016).

In order to test our method, we tested it in a case of submersion with first responders in France: the SDIS 44. We show how our method helps determine the preferences of the decision maker and how it can be applied in projections under IPCC's RCP 8.5 scenario in the Pays-de-la-Loire region, following a storm of similar magnitude to Xynthia to the city of Batz-sur-mer, one of the cities of interest of the MOGEC project.Finally, we draw some conclusions from our work and call the development of a tool directly accessible for geographers allowing them to use this method in other contexts.

2. Related works

In cartography, it is common to develop reasoning by analyzing the interactions between multiple parameters. This study aims to highlight the intersections between various issues that may be affected during a marine submersion crisis (i.e., coastal flooding due to sea incursions), with a focus on identifying priority intervention areas.

Numerous methods already exist for analyzing a given territory, often through the early identification of vulnerable zones. The 2010 Xynthia storm in France led to widespread awareness and necessitated the implementation of new risk management strategies. For instance, (Creach, 2017) developed a methodology to identify sensitive buildings and infrastructure in crisis scenarios, thereby allowing the designation of priority intervention sectors and reducing the vulnerability of exposed assets. The V.I.E index is based on several criteria, such as proximity to refuge zones or potential water depth, and results in an intrinsic vulnerability score ranging from A to D. This tool is calculated using a weighted aggregation method.

Several researchers have applied similar mathematical frameworks to adapt land-use planning to specific needs. For example, (Pre´vil et al., 2003) propose a comparable approach that considers local parameters to assign different weights, integrating all relevant domains (e.g., land use, environment). This approach, referred to as SIG-AMCD, involves the construction of multiple scenarios evaluated against different criteria. Similarly, (Rufat, 2007) presents a methodology for mapping urban risks through the definition of vulnerability indicators, automatic classification of vulnerability levels, and their cartographic application. (Arnaud et al., 2023) adopt a similar strategy for assessing coastal erosion, involving variable selection, data collection, risk component calculation, and weighted aggregation to produce final maps. (Bengoufa et al., 2021) also follow a related methodology, using five physical and socio-economic variables, subsequently weighted and mapped. (Lagadec et al., 2018) offer a multi-criteria analysis for runoff risk management and railway infrastructure. Their approach, while distinct, relies on statistical indicators such as the chi-squared test, the definition of three hazard criteria, and the IRIP mapping method (Dehotin and Breil, 2011). Vulnerability criteria are then defined and cross-analyzed with hazard data through specific statistical computations tailored to the theme.

While these methodologies ensure strong replicability and interpretability of results, they often fail to account for interactions between criteria, since each criterion is assessed independently of potential coupling effects. To our knowledge, few studies have explored such interactions. Nevertheless, some works have employed the Choquet integral (Grabisch and Labreuche, 2010) in risk assessment frameworks. For example, (He and Weng, 2021) use the Choquet integral combined with linear regression to assess coupling risks in chemical hazards. However, their approach mainly targets multi-hazard scenarios and not the analysis of multi-dimensional interactions. Moreover, their analysis relies on historical disaster data to model coupling effects via regression, thus limiting its applicability to contexts where such data are available. (Sun et al., 2022) uses a similar approach, using the Choquet integral to process historical data on the Yangtze river and compute redundancy and interaction within data. (Moradi et al., 2017) utilize the Choquet integral to identify interactions among physical, social, and systemic criteria, gathering input from around fifty experts and relying on the definition of Shapley indices and interaction indices. While precise, their method requires a large number of expert opinions, making it less easily replicable and often less sensitive to local specificities.

In this study, we propose a novel methodology based on the Choquet integral, incorporating local expert input through an extension of the MACBETH method (Bana e Costa and Vansnick, 1999) to elicit and formalize expert preferences. Following the logic of (Moradi et al., 2017), we rely on existing cartographic layers and professional expertise to construct a risk map. However, by engaging with a local expert authority—in this case, SDIS 44 (the fire and rescue service of Loire-Atlantique)—our approach achieves both higher replicability and greater contextual relevance.

3. Assessing risk with Choquet integral and MACBETH

In this section, we present our novel tool for risk mapping based on 2-additive Choquet integral and MACBETH approach. We fist introduce the Choquet integral and its 2-additive variant. We then present MACBETH and how it can be used for the elicitation of the capacities of the 2-additive Choquet integral. Finally, we present how this approach can be used for risk mapping

3.1 Choquet integral

3.1.1 Choquet integral for MCDA First, let us set the problem. We have N criteria, in our case, it could be the accessibility of the zone, the criticality of the assets and the submersion likelihood (linked to the altitude of the location being considered) We therefore have $N = 3$, submersion likelihood being associated to 1, asset criticality to 2 and accessibility to 3. For the moment, we suppose that all these criteria are set between 0 and 1. We will see later on in this article how to get them. Before defining the Choquet integral, we need to define capacities:

Definition 1 (Capacity). We call **capacity** a function $\mu : 2^N \rightarrow [0, 1]$ such that: $\mu(\emptyset) = 0$, $\mu(N) = 1$ and $\forall (A, B) \in 2^N \times 2^N, A \subseteq B \implies \mu(A) \leq \mu(B)$(monotony)

The capacity can be defined as a kind of satisfaction function: if no criteria are fulfilled, the function equals 0, if all of them are fulfilled, the function equals 1, and for any set of criteria, fulfilling additional criteria while keeping every already fulfilled criteria still fulfilled improves the satisfaction.

Using a capacity, it is possible to define something that is not possible with a weighted sum. Let us take the aforementioned example. Suppose you want to represent the following situation: if the likelihood of submersion is null, then the utility is 1, whatever the accessibility and the buildings, but if it is not the case, the utility would be 0.4 for low-priority assets and perfect accessibility, 0.3 for low-priority assets and bad accessibility and 0.1 pour high-priority assets and perfect accessibility. It is not possible to represent this situation with a weighted sum. The first statement implies that the coefficient associated with the likelihood of submersion, while the other ones implies non-zero coefficients for other criteria. Representing the situation with a capacity, we get: $\mu(1, 0, 0) = \mu(1, 1, 1) = 1$, $\mu(0, 1, 1) = 0.4$, $\mu(0, 1, 0) = 0.3$, $\mu(0, 0, 1) = 0.1$, $\mu(0, 0, 0) = 0$.

A capacity defines a satisfaction or utility level when the criteria are either fulfilled or not at all. Now, we would like to find a function that can be applied to fuzzy variables representing criteria that are neither totally fulfilled nor not fulfilled at all, and we would like this function to correspond to the capacity where the latter is defined. This function is the Choquet integral. To define the Choquet integral, we therefore need to define a set of variables in $[0, 1]$ representing to which extent the criteria are fulfilled. We note these variables $f_i \in [0, 1]^N$. In our example, f_1 corresponds to the likelihood of submersion, f_2 the criticality of the assets in the cell, and f_3 its accessibility. We can now define the Choquet integral.

Definition 2 (Choquet integral)**.** Let $f \in [0, 1]^N$ and $\mu : 2^N \rightarrow [0, 1]$ a capacity over N

Let order the values of f in descending order, and define σ the corresponding permutation function: $f_{\sigma(1)} \geq f_{\sigma(2)} \geq \cdots \geq f_{\sigma(N)}$. We also note $f_{\sigma(0)} = 0$

The Choquet integral of f w.r.t μ $C_\mu : [0, 1]^N \rightarrow [0, 1]$ can be defined as:

$$C_\mu(f) = \sum_{i=1}^{N} \left(f_{\sigma(i)} - f_{\sigma(i-1)}\right) \cdot \mu\left(\bigcup_{k=i}^{N} \{\sigma(k)\}\right) \quad (1)$$

Defining the Choquet integral is therefore totally determined by the corresponding capacity. In multi-criteria decision aid, the goal is for a decision maker (DM) to determine the parameters of a function representing their preferences. In our example, the goal would therefore be for the DM to determine the capacity. The main issue is that, beyond the method to do so (that we will describe lower), the number of coefficients to elicit is 2^N. When there are only 3 criteria, we only get 8 coefficients to determine, but with more criteria, this number becomes incalculable. To avoid this issue, we restrain to the 2-additive Choquet integral.

3.1.2 2-additive Choquet integral and trees 2-additive Choquet integral is a specific form of Choquet integral. While Choquet integral can be thought of as a weighted sum of the value of criteria and the value of their interaction, 2-additive Choquet integral defines any interaction between 3 criteria or more as null, reducing significantly the number of coefficients to determine.

To define more properly the -additive Choquet integral, we first need to define the interaction indexes.

Definition 3 (Interaction index)**.** Interaction among a set of criteria A can be defined as:

$$I^\mu_{\{A\}} = \sum_{K \in 2^{N \setminus A}} \frac{(N - |K| - |A|)!|K|!}{(N - |A| + 1)!} \times \sum_{L \in 2^A} (-1)^{|A|-|L|} \mu(K \cup L) \quad (2)$$

In order to better understand the interaction indexes and because it is of interest for the rest of our study, we apply this definition to $|A| = 2$ and we define interaction index between i and j and we write I_{ij} the following index:

$$I^\mu_{ij} = \sum_{A \in 2^{N \setminus \{i,j\}}} \frac{(n - |A| - 2)!|A|!}{(n - 1)!} \times \mu(A \cup \{i, j\}) - \mu(A \cup \{i\}) - \mu(A \cup \{j\}) + \mu(A) \quad (3)$$

The 2-additive Choquet integral is defined by additional constraints on the capacity:

$$\forall A \in 2^N, |A| > 2 \implies I_A = 0 \quad (4)$$

This relation means that there is no direct interaction among a set of criteria besides pairs of criteria. In the definition of the capacity, it also means that it is possible to determine the values of $\mu(A)_{|A|>2}$ using the eq. (4). While this constraint reduces the possible functions, it also decreases the number of coefficients to $\frac{n \cdot (n+1)}{2} - 1$ which is much more manageable for decision makers. In our example, it means, for example, that there is no direct interaction between submersion likelihood, criticality of assets and accessibility all at once. Interaction still may exist between submersion likelihood and criticality of assets, between submersion likelihood and accessibility or between criticality of assets ans accessibility.

However, even $\frac{n \cdot (n+1)}{2} - 1$ coefficients may be a lot to determine. Moreover, it may happen that some criteria do not directly interact with each other. In our case the criticality of the assets is actually defined both by the kind of asset and the height of the last floor, which defines if a person can seek refuge at that floor. And the accessibility is actually defined by the width of the roads and their type. While overall, the interaction between accessibility and criticality of assets makes sense, it is not the case for the interaction between the type of road and the height of the last floor of the buildings. To deal with these two issues, it is possible, as in MYRIAD (Labreuche and Le Hue´de´, 2005) to build a tree that will define intermediary criteria, which are aggregations of lower-level criteria, and which can be aggregated themselves. The resulting tree of the aforementioned case is represented on fig. 1. On this figure, we can see that the road width and road type are aggregated through a Choquet integral; similarly, the asset type and last floor height are aggregated into a higher-degree criterion: asset criticality. Finally, the overall utility is an aggregation of accessibility, submersion likelihood and asset criticality. The only remaining step is the definition of the criteria (in blue) from the metrics (in pink).

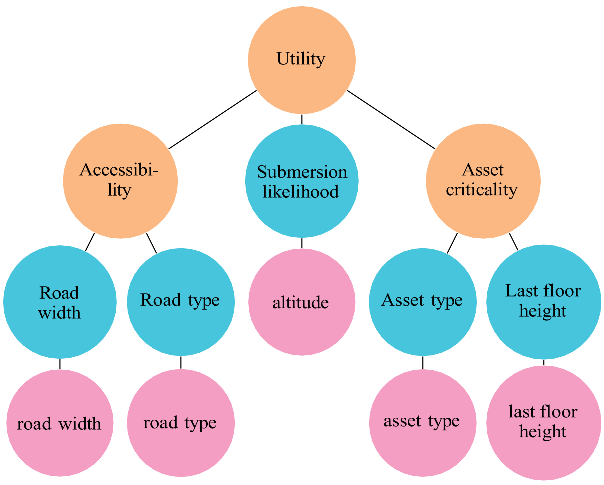

Figure 1. Decision tree: the metrics gotten from the data are in pink, the criteria are in blue, and the aggregations are in orange

We consider two cases: the quantitative metrics, and the qualitative ones. For the quantitative metrics, the transformation into a criterion is performed by a piecewise-linear function. This function goes from the set of possible values for the quantitative value to $[0, 1]$. For qualitative metrics –including ordinal and categorical ones– each of the values is associated to a value in $[0, 1]$.

The evaluation process can be retrieved from fig. 1: the road width, road type, altitude, asset type and last floor height is obtained from data. Each of them is transformed in a value in $[0, 1]$ by a function. then, Road width and Road type are aggregated by a Choquet integral, and the Asset type and last floor height also. Then the resulting values, both in $[0, 1]$ are aggregated with the Submersion likelihood into a value in $[0, 1]$. In the following subsection, we see both how the values of the coefficients of the Choquet integral are computed, and how the criteria are computed from the metrics using the MACBETH method.

3.2 MACBETH

The MACBETH method, defined in (Bana e Costa and Vansnick, 1999, Bana e Costa et al., 2016) is a method used to determine the preference levels of a decision maker. In this section, we discuss this method and see how it can be applied to the computation of criteria from metrics and the Choquet integral.

3.2.1 The MACBETH method The MACBETH method has initially been designed to define the preferences among a set of alternatives. This use case corresponds perfectly to the computation of a criterion from a qualitative metric. Let us take the asset type, and call it $A = \{\alpha_1, \dots \alpha_{|A|}\}$ be this set –in our case, retrieved from the IGN data: residential, commercial, industrial, sportive, agricultural, religious, annex, undifferentiated and $|A| = 8$.

A possible approach would be to directly ask to the utility of each of the alternatives to the DM, but this is actually very hard, in particular when there are multiple alternatives. MACBETH proposes an alternative by letting the DM propose comparisons between the alternatives. In order to represent different levels of preference of the DM, they are proposed to define their level of preference. The next question is: which is the number of level of preferences proposed to the DM? The goal is for the DM to be able to differentiate between two levels of preferences –consequently there should not be too many levels– but also to express fully their preferences –consequently, there should be enough levels. MACBETH proposes 6 levels of preference, plus an indifference level. The corresponding labels are: very weak, weak, moderate, strong, very strong and extreme.

A remaining issue is the uncertainty of the DM. They may not be sure of the preference between two alternatives, as these alternatives may cover subcategories, or their preference may vary depending on the context. To deal with this issue, the DM is proposed to express their preferences as a set of possible preferences: for instance, a DM may say that they prefer that an agricultural asset be the object of a submersion rather than a residential asset with a preference varying from moderate to very strong.

In order to represent the preferences of the DM, we use 7 binary relations $C_0, C_1, \dots, C_6$. Let $(\alpha, \beta) \in A^2$ be two alternatives. Then $(\alpha, \beta) \in C_0$ means that the DM is indifferent to α and β while $\forall k \in \llbracket 1, 6 \rrbracket$, $(\alpha, \beta) \in C_k$ means that k is one of the possible preference levels for the DM between α and β. If we call α the residential assets and β the agricultural assets, we can therefore write $(\alpha, \beta) \in C_3 \cup C_4 \cup C_5$.

The goal of the MACBETH method is to allocate a value function v to the alternatives. By convention, it is considered that the least desirable alternative α^- gets a utility of 0: $v(\alpha^-) = 0$. We also call α^+ the most desirable item. To find the values of the alternatives (other than α^-) we use a soler to solve the following linear problem:

$$\min v(\alpha^+) - v(\alpha^-) \text{ such that} \tag{5}$$

(1) $v(\alpha^-) = 0$

(2) $\forall (\alpha, \beta) \in C_0, v(\alpha) = v(\beta)$

(3) $\forall (lb, ub) \in \llbracket 1, 6 \rrbracket^2 : ub \geq lb$

$$(\alpha, \beta) \in \bigcup_{k=lb}^{ub} C_k \implies v(\alpha) - v(\beta) \geq lb$$

(4) $\forall (lb, lb', ub, ub') \in \llbracket 1, 6 \rrbracket^4 :$

$$ub \geq lb, ub' \geq lb', lb > ub',$$

$$(\alpha, \beta) \in \bigcup_{k=lb}^{ub} C_k, (\alpha', \beta') \in \bigcup_{k'=lb'}^{ub'} C_{k'}$$

$$\implies v(\alpha) - v(\beta) \geq v(\alpha') - v(\beta') + lb - ub'$$

The condition (3) states that if an alternative α is preferred with a level of at least lb to another one β, then the difference $v(\alpha) - v(\beta)$ should be at least lb. The condition 4 states that if α is preferred to β with a level of at least lb and α' is preferred to lb with a level of at most $ub' < lb$ then, the difference between $v(\alpha) - v(\beta)$ and $v(\alpha') - v(\beta')$ should be at least $lb - ub'$ to represent the difference between the preference level between on the one hand α and β and on the other α' and β'.

Once the value function is computed, the values are normalized between 0 and 1, by dividing all of them by $v(\alpha^+)$

3.2.2 Using MACBETH to compute criteria from quantitative metrics The extension to quantitative metrics is pretty easy. The DM defines several intermediate values that will be used as alternatives. The quantitative values are then ordered either in ascending or descending values, and then a linear interpolation defines the values for all values between consecutive values.

3.2.3 MACBETH for Choquet integral This method is based on (Labreuche and Grabisch, 2003). As explained in section 3.1.2, determining the 2-additive Choquet integral only requires to determine the value of $\frac{n \cdot (n+1)}{2} - 1$ coefficients: $\mu(\{i\})_{i \leq N}$ and $\mu(\{i, j\})_{i \leq j \leq N}$. We therefore ask the DM to compare situations where one or two criteria are totally fulfilled. In order to make it easier for the DM to express their preferences, we also transcribe these situation in a more realistic context. We also add constraints on the problem regarding the capacities (monotonicity constraints, $\forall (i, j) \in \llbracket 1, N \rrbracket^2, \mu(\{i, j\}) \geq \mu(\{i\})$, and bounds) and constraints on the 2-additivity computed using (Mayag et al., 2008).

3.3 Risk mapping

Our approach is organized as follows. First, the Choquet integral tree is built with the decision maker. We first build gather the data we have, that will be used as the metrics of the Choquet integral. The DM are then consulted to know which of the data is relevant to assess the risk of natural hazard on the location. Then, using MACBETH, the criteria coefficients and Choquet integrals capacities are calculated. The map of the studied area is then divided into a grid. For each cell of the grid, we extract the relevant metrics in order to compute the Choquet integral. The overall process of the computation is described on fig. 2. The process produces a new file that can be interpreted by a GIS tool.

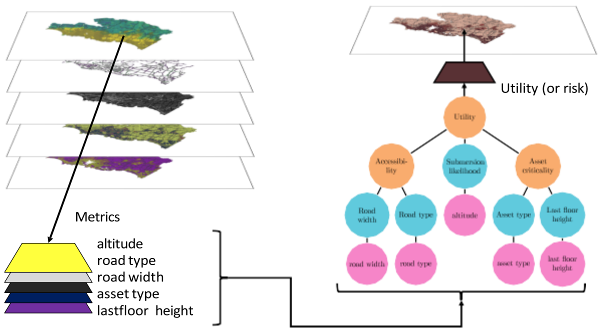

Figure 2. representation of the overall process of our risk mapping module: based on the data we have, we extract each metric for each cell. Based on the preferences expressed by the DM, the Choquet integral is calculated for each cell, and stored in a new shape file.

4. Flood risk assessment in Batz-sur-mer

4.1 Setup

Our approach has been tested on the city of Batz-sur-mer, in Loire Atlantique, France, and is related to the MOGEC project (Savary et al., 2024). This site has been chosen due to its specific features. The city is near salt marshes in its North, which have a relatively low topography (between 0 and 5 m). The rocky coastlines to the south act as natural barriers, providing a protective function for the area against marine submersion. 54% of the municipality lies within a submersion-prone area. The municipality also has very different assets, including two schools. The population variation is also a challenge for the decision makers. While the population is 2,823 the population can rise to 18,000 during the peak summer season, making it hard to determine the importance of assets. The risk of submersion is pretty high, with 500 residential buildings and 11 business buildings being vulnerable to flooding as they are located in low-lying coastal areas.

4.2 Decision makers interviews

In order to evaluate our approach, we relied on the emergency services of the department, the SDIS 44. We planned several meetings with them, and got the expertise from Cdt David Regnouf. We wen through the MACBETH process with him. We first worked on the tree, and got to the tree represented in fig. 1. We have 5 different kinds of metrics: three of them are quantitative: altitude, road width, and last floor height. We have two qualitative metrics: the building type, already presented above, and the road type, with seven different values.

We then asked him his preferences for each of the criteria and for each aggregation. We got the preferences of the DM directly during a meeting for most of the metrics. However, for time reasons, we sent several questions afterwards, asking him to fill some of the values of the overall aggregation, and also to order and express preference levels for the asset types.

4.3 Computation

The code for our study can be found on https://gitlab.univ-nantes.fr/buron-c-1/lear. The code is written in python, with the use of the pyshp library (Lawhead and Bahgat, 2022).

The metrics are retrieved using the IGN database. We made several choices during the implementation: first, regarding the scale, we chose to use a multiple-scale cartography. The reason is that there are wide areas with few differences in Batz-sur-mer, in particular in places where there are salt marshes. The cells are 50 meters wide in areas without large roads, and 25 meters wide near roads and buildings. We faced another difficulty. Even in dense area with cells of 25 meters, we still have multiple assets of various types, and various roads of various size. We decided to keep the larger roads, considering that regarding the width of our cells, any road would let the emergency services get to any person within the cell. For the building, we chose the most critical one (according to the emergency services) and the corresponding last floor height. Finally, considering that people could go up to take refuge, we decided to keep the highest altitude of each cell. We used QGis to handle the data.

Once the data is retrieved, the answers of the Cdt Regnouf were processed through the MACBETH process presented in the section 3.2. For linear programming, we rely on the scipy library, and the HiGHS solver (Galabova, 2023). The resulting values are stored in a new shapefile using the pyshp library. The resulting file can be opened with QGis.

5. Results

The result of our approach can be seen on fig. 3. The first notable observation is that the overall utility values are relatively high. This can be attributed to the urban layout of Batz-sur-Mer, where the majority of critical assets are situated on elevated sections of the town, including hamlets located away from the town center (represented by cells with smaller areas). Similarly, these cells often include relatively wide roads, with one or two lanes, which facilitate evacuation. As a result, the criteria tend to balance each other out, because of the substitutability of altitude and asset criticality –high submersion likelihood with low priority assets or low submersion likelihood with high priority assets being considered not too critical.

Nevertheless, residential and commercial districts –particularly in the southwest– exhibit lower utility values, ranging between 0.6 and 0.7, which are not offset by the number of building stories in these areas. This effect is especially visible when examining the zones situated between inhabited areas. These zones are at the same elevation as residential zones but demonstrate significantly higher utility values, even higher than those of the salt marshes (with utility scores between 0.73 and 0.77). Although the asset presence in the marshes is minimal, the associated likelihood remains higher due to the lower elevation compared to the intermediary zones. That said, there are areas containing assets with very high utility values, though these assets themselves are not considered highly critical.

The influence of the road network is also evident: major roads tend to exhibit slightly higher utility values than the surrounding cells. However, roads in isolation have limited impact, which explains the modest magnitude of these differences.

Low utility values (below 0.5) are observed in areas hosting significant assets that are located in the lowest-lying zones. These buildings mark both the topographical boundary between elevated and lower-lying areas and the urban edge adjacent to the salt marshes. Some critical buildings –particularly residential

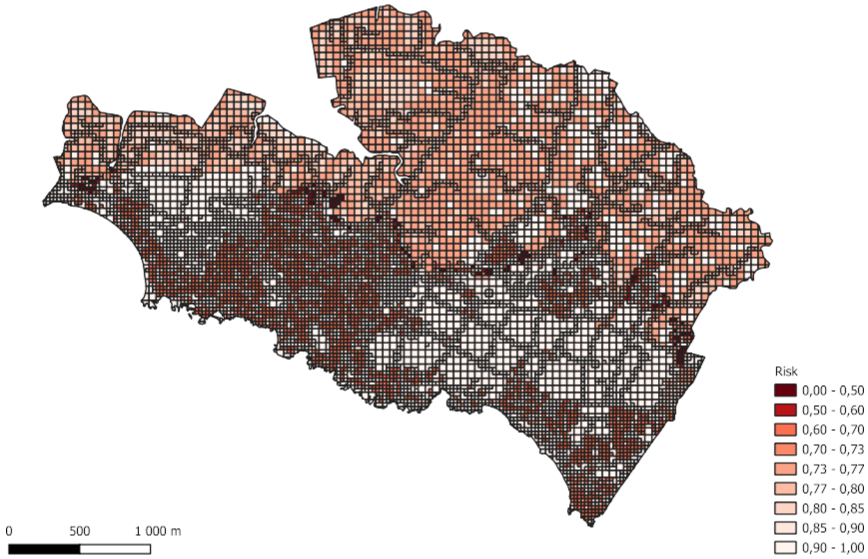

Figure 3. Computed vulnerability based on the Choquet integral. Lower score means higher vulnerability.

ones– are located in these zones, which accounts for their classification as critical.

Overall, the map effectively reflects the preferences of emergency services. It highlights their prioritization of areas containing critical assets that are also at risk due to low elevation. The intersection of asset criticality and elevation –two criteria whose interaction is characterized by a significant negative interaction index (-0.36)– allows for the identification of high priority areas of intervention: critical assets situated in low-lying areas. Critical assets in higher areas are considered less vulnerable but are still detected, as much as low-lying areas where individuals may be rapidly endangered by rising waters.

6. Conclusion

In this article, we presented a new tool for assessing natural disaster risks, based on the Choquet integral and an adaptation of the MACBETH method tailored to this context. We demonstrated how the Choquet integral enables the modeling of interactions between criteria, thereby allowing for the representation of preferences that cannot be captured using a traditional weighted sum. To facilitate the decision-maker's ability to parameterize the model, we restricted the function to the 2-additive Choquet integral.

We then introduced the MACBETH method and showed how it can be applied to the elicitation of the decision-maker's preferences –both for deriving criteria scores from measurable indicators and for determining the parameters of the Choquet integral. Finally, we illustrated how these decision-support tools can be effectively integrated into geographic mapping.

This methodology was applied to the case of submersion in Batz-sur-Mer. We conducted interviews with the local emergency services (SDIS 44) to elicit their preferences, which allowed us to define a vulnerability factor incorporating all relevant criteria. We then showed how the resulting map, thanks to the non-linear properties of the Choquet integral, offers a faithful representation of the emergency services' prioritization logic.

Our future work includes the development of a more intuitive and user-friendly tool, enabling decision-makers to interact with the system through a graphical user interface, even during the preference elicitation phase. We also aim to incorporate visual indicators of criterion importance (Shapley indices), as well as the significance of interactions between criteria. Additional explainability measures –such as those proposed by (Labreuche and Fossier, 2018)– could further enhance the interpretability of the model. Regarding the model, additional criteria could be added to enrich it, like calculated time of submersion depending on the location of the assets.

Lastly, our current approach lacks a mechanism to account for the presence of multiple buildings within a single spatial unit (even within 25 meters wide cells). To address this, we propose drawing on the work of (Labreuche et al., 2015), which offers potential extensions suited to this challenge.

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