TOPOLOGICAL RELATIONSHIPS IN R^3 FOR 3D CADASTRE

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ABSTRACT:

3D cadastre is necessary for accurately representing real-world parcels that not only exist on land but also above and below ground. The ability to cater to the physical and legal attributes of these 3D parcels is important as it affects the Rights, Restrictions and Responsibilities (RRRs). Tasks such as modification and analysis of 3D parcels also require topological information in addition to the geometrical properties of a 3D parcel. Topological properties describe the connectivity information between objects. In terms of the 3D cadastre, the intrinsic topology is used to ensure the spatial objects that make up a 3D solid are valid, while extrinsic topology is used to determine adjacent 3D solids or overlapping parcels. This paper attempted to define topological relationships in 3D space specifically for the use of 3D cadastre. 3D to 3D solid and 2D to 2D surface topological interactions were defined and proved using the Dimensionally Extended Nine Intersection Model (DE-9IM). The “meet (touches)” and “overlaps” topological relationships were found to be the main topological relationship used in the 3D cadastre.

1. INTRODUCTION

As cities continue to expand and grow, occupied space or parcels have taken on a vertical component. As a result, 3D cadastre parcels can exist above, below and on land, which in turn affects the Rights, Restrictions and Responsibilities (RRRs) of each parcel. The representation of parcels in 2D with their RRRs is not sufficient for tasks which involve complex or overlapping parcels (van Oosterom et al., 2018). Management, modification and analysis of 3D parcels necessitate accurate representation of parcels in 3D space, including their legal and physical attributes. Parallel to the Land Administration Domain Models (LADM), the use of CityGML, IFC, InfraGML, BIM and GIS were also found to be suitable for 3D cadastre purposes (Sürmeneli, Koeva, & Alkan, 2022). Further development of CityGML 3.0 has also included a new subclass, “AbstractBuildingSubdivision” that describes 3D cadastral information such as “BuildingUnit” and “Storey” in addition to the existing “Building” class (Kutzner, Chaturvedi, & Kolbe, 2020).

Geometrical properties are crucial to describing the shape, dimension and location of the 3D parcel. Topology as an accompaniment to the geometry of the 3D parcel is also important to maintain connectivity information. Topology can be defined as the study of topological transformations and the properties that remain unchanged by changes to the space (Worboys & Duckham, 2004). In other words, topological properties of objects such as adjacencies, connectivity and containment remain unchanged (Ellul & Haklay, 2006; McDonnell & Kemp, 1995). Apart from geometric properties that describe the shape of the object, topological properties are also important in explaining the relatedness of objects within the space. Therefore, topological properties of an object are properties that remain unchanged despite transformations of the space and define the qualitative properties of the object in terms of relationships to other objects. Further reading can be done based on the results of the previous research (Salleh & Ujang, 2018; Ujang, Anton Castro, & Azri, 2019).

Advances in data acquisition have provided detailed and accurate 3D spatial data as the basis for any task. Topology as a spatial property also gives value to 3D spatial data by maintaining 3D connectivity, adjacency and containment information. The preservation of 3D topological properties of a spatial object can be implemented using several approaches such as topological models, data structures and topological rules. A topological model is a schema that specifies the representation of topological interactions or characteristics of spatial objects (Ghawana & Zlatanova, 2012; Lee & Kwan, 2005). The topological model, as a schema, explains topological relationships in the absence of a physical data structure that explicitly stores such properties (Arroyo Ohori, Ledoux, & Stoter, 2015). As a result, a topological model may be utilised to preserve topological relationships in a simple and lightweight method. A topological data structure is a physical structure that stores and retains the topological properties of objects (Arroyo Ohori et al., 2015). In turn, the maintenance of comprehensive topological properties requires ample storage space and extensive computation capabilities. Topological rules are the definitions of valid topological interactions between objects. This enables topological relationships between objects to be established based on the objects' fulfilment of requirements. Topological rules ensure that objects are topologically valid without the complications of a topological data structure (Martinez-Llario, Coll, Núñez-Andrés, & Femenia-Ribera, 2017). As an outcome, topological rules can be used to connect geometrical and topological models when geometrical models alone are unable to satisfy spatial reasoning requirements (Solihin, Eastman, & Lee, 2017).

This paper mainly focuses on defining and describing 3D topological relationships for 3D cadastre purposes. Section 2 presents related studies that utilise topological information for 3D cadastre. The methodology will be described in Section 3. Subsequently, Section 4 presents the definitions of topological relationships are detailed in Section 3. Finally, Section 5 puts forth the conclusion of this study.

2. RELATED STUDIES

Since urban development should be depicted in a 3D environment, topology information is necessary. Virtual campuses (Salleh, Ujang, & Azri, 2021), smart city sensors (S Azri, Ujang, & Rahman, 2019), and handling 3D data for urban applications (Azri, Ujang, Castro, Rahman, & Mioc, 2016; Suhaibah Azri, Ujang, Rahman, Anton, & Mioc, 2014) require information on the spatial relationships between objects. In terms of 3D geometry, a 3D parcel as a 3D primitive object can be defined as one close polyhedron which consists of a set of connected faces (Kazar, Kothuri, van Oosterom, & Ravada, 2008). Nonetheless, 3D parcels can have complex shapes and surfaces. For the uses of 3D cadastre, a valid 3D parcel is allowed to have holes where boundary faces may touch each other, internally connect, or self-touch as long as the interior of the volumetric object remains connected (Kazar et al., 2008). The complexities of the geometrical properties can be accurately described with the accompaniment of topology. This provides the fundamental information required for more complex tasks such as parcel boundary utilisation for 3D modelling, ensuring valid and topologically consistent 3D objects, and efficient topological queries (van Oosterom et al., 2018). In recent studies, the topology framework employed for 3D cadastre can be divided into two main approaches, which are the classification of topological relationships and the implementation of topological structures.

Various methods can be implemented to classify topological relationships for 3D cadastre. A study by Emamgholian, Taleai, and Shojaei (2021) utilised a close proximity analysis to determine topological relationships between adjacent 3D units within a building. This includes adjacent horizontal and vertical 3D units in high-rise buildings, as shown in Figure 1.

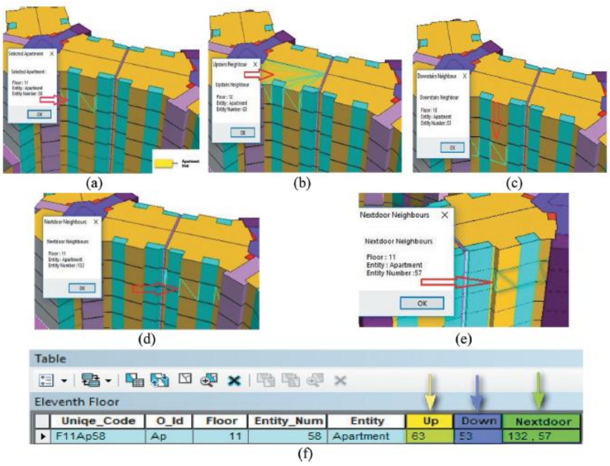


Figure 1. 3D unit with queried adjacent units based on close proximity analysis (Emamgholian et al., 2021).

Legal boundaries of 3D cadastral units that consist of interior, exterior and median boundaries can also be determined based on touches, overlaps and covers topological relationships. The legal boundaries are similar to the variables of the 9-intersection model (9IM), which denotes intersections between the boundaries, interiors and exteriors of two objects. Based on that, Barzegar, Rajabifard, Kalantari, and Atazadeh (2020) implemented a ray test which propagates vertical and horizontal rays from the center of the 3D unit to classify the topological relationships between the 3D units and determine the legal boundaries, as shown in Figure 2.

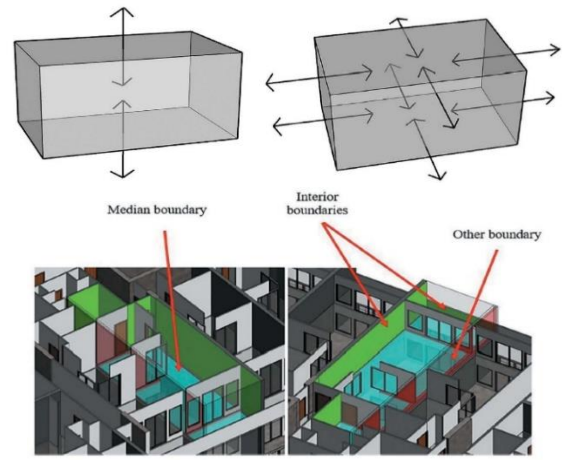


Figure 2. Propagated vertical and horizontal rays with resulting legal boundaries of 3D units (Barzegar et al., 2020).

On the other hand, topological structures are also implemented for the 3D cadastre topology framework. Topological structures define how topological properties are physically stored within the framework. Jaljolie, Riekkinen, and Dalyot (2021) utilised a hierarchical 3D volumetric parcel structure (3DVP) which consists of a 3D face, 3D node and 3D line, as shown in Figure 3. The hierarchical structure facilitates more complex 3D capabilities for 3D cadastre, such as 3D parcel subdivision, volume calculation and 3D buffer analysis (Jaljolie et al., 2021).

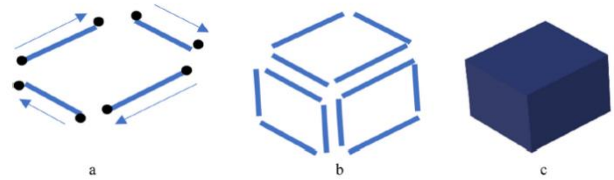


Figure 3. 3D volumetric parcel structure composed of (a) 3D nodes and lines, (b) 3D face and (c) 3D parcel (Jaljolie et al., 2021).

Parallel to previous studies, the role of topology in assisting 3D cadastre can be expressed as intrinsic and extrinsic topology. Intrinsic topology describes the internal connectivity within a 3D object, while extrinsic topology describes the connectivity or relationship between 3D objects (Knoth, Atazadeh, & Rajabifard, 2020). Therefore, intrinsic topological capabilities are required for the validation of 3D objects. Meanwhile, extrinsic topology assists in the deduction of adjacency information between 3D objects as well as the detection of overlaps or crosses.

3. METHODOLOGY

The topological interactions between 3D objects focused on in this study are 3D solid to the 3D solid and 2D surface to 2D surface. The topological relationships are defined based on intersections of interiors, boundaries and exteriors of spatial objects in 3D space (R^3) as denoted by the Dimensionally-Extended Nine Intersection model (DE-9IM). The intersection matrix for DE-9IM is shown in Equation 1.

$$R_{DE-9IM}(A,B) = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^- \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^o & A^- \cap \partial B & A^- \cap B^- \end{bmatrix} \qquad (1)$$

where A^o = Interior of A
∂A = Boundary of A

A^- = Exterior of A
B^o = Interior of B
∂B = Boundary of B
B^- = Exterior of B

Table 1 describes the interiors, boundaries and exteriors of 2D and 3D objects in R^3. The following sub-sections present the definitions for topological relationships in 3D space.

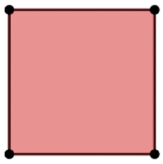
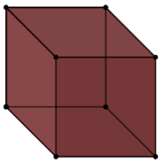
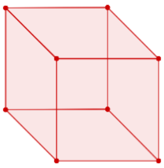
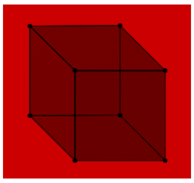
Object (Dimension)	Interior	Boundary	Exterior
Surface (2D)	 (surface)	 (vertices and lines)	 (non-empty)
Volume (3D)	 (volume)	 (vertices, lines and surfaces)	 (non-empty)

Table 1. Interior, boundary and exterior of spatial objects in R^3.

4. DEFINITIONS FOR TOPOLOGICAL RELATIONSHIPS IN R^3 FOR 3D CADASTRE

3D cadastre represents individual building units or 3D parcels that have shared surfaces or adjacencies, which are denoted as the topological relations crucial for spatial analysis. Fundamentally, 3D objects must be defined as valid solids using geometric rules to ensure geometrically sound data. Therefore, topological relations should also be defined as valid interactions between 3D objects.

4.1 3D to 3D Topological Relationships

In terms of usage within the 3D cadastre, topological relationships between 3D solids are "meet (touches)" and "overlaps". This fulfils the requirement of retrieving adjacency information between 3D parcels. The "meet (touches)" relationship can be used to determine neighbouring 3D solids. Figure 4 depicts two 3D solids that have a "meet" and "touches" topological relationship at a 2D surface.

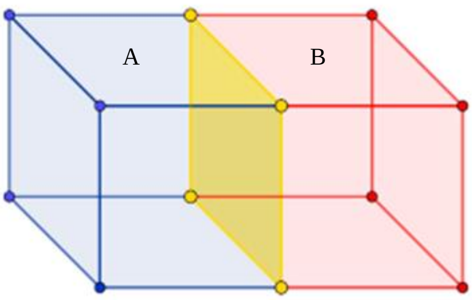


Figure 4. 3D to 3D "meet (touches)" topological relationship.

The 3D to 3D "meet (touches)" topological relationship can be defined as follows;

Condition: Two 3D solids "meet (touches)" when no intersection occurs between 3D volume interiors but intersects at a common boundary. The common boundary of the 3D solids can be either 2D surfaces, 1D lines or 0D vertices.

The intersection matrix for the case in Figure 4 is expressed below;

Proof:

$$\dim(A,B) = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^- \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^o & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 3 \\ -1 & 2 & 2 \\ 3 & 2 & \neg\emptyset \end{bmatrix}$$

where -1 = no intersection
2 = intersection at 2D surface
3 = intersection at 3D volume
$\neg\emptyset$ = has intersection

Based on the intersection matrix above, no intersections occur between the interiors of the objects, and an intersection of a common 2D surface occurs at the boundary of both objects.
Another common occurrence in 3D cadastre is overlapping 3D parcels. Figure 5 displays two 3D solids that overlap.

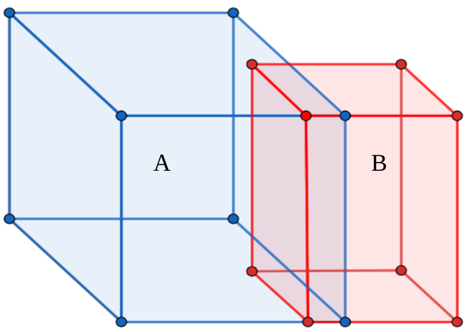


Figure 5. 3D to 3D "overlaps" topological relationship.

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The 3D to 3D "overlaps" topological relationship can be defined as follows;

Condition:

Two 3D solids "overlaps" when the 3D volume interiors of both objects intersect and also having interiors that intersect with the exterior of the other object.

The intersection matrix for the case in Figure 5 is expressed below;

Proof:

$$\dim(A,B) = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^- \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^o & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & \neg\emptyset \end{bmatrix}$$

where 2 = intersection at 2D surface
3 = intersection at 3D volume
$\neg\emptyset$ = has intersection

Based on the intersection matrix above, 3D volume intersections occur between the interiors of both objects.

4.2 2D to 2D Topological Relationships

As mentioned in the previous section, intrinsic topology is crucial for the internal validation of a legal 3D parcel. The internal consistency of a 3D object necessitates the surfaces to be closed polyhedrons without any self-intersecting surfaces. Closed surfaces are required to adhere to the "meet (touches)" topological relationship. Figure 6 displays 2D surfaces that "meet (touches)" at a common boundary.

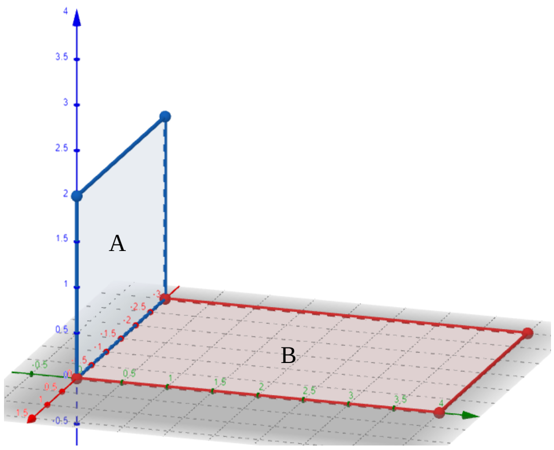

Figure 6. 2D to 2D "meet (touches)" topological relationship with a common boundary.

The 2D to 2D "meet (touches)" topological relationship which intersects at a common boundary can be defined as follows;

Condition: 2D surfaces "meet (touches)" when the interior surfaces do not intersect. The 2D surfaces must also have a common boundary at either 1D line or 0D vertices.

The intersection matrix for the case in Figure 6 is expressed below;

Proof:

$$\dim(A,B) = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^- \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^o & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 2 \\ -1 & 1 & 1 \\ 2 & 1 & \neg\emptyset \end{bmatrix}$$

where -1 = no intersection
1 = intersection at 1D line
2 = intersection at 2D surface
$\neg\emptyset$ = has intersection

Based on the previous intersection matrix, no intersections occur between the interiors. Nonetheless, the two surfaces share a common 1D line boundary.

Besides that, the 2D surfaces also cannot self-intersect in order to ensure internal topological consistency. This occurs when a boundary of a 2D surface intersects with the interior of the other 2D surface. The detection of self-intersecting surfaces also relies on the 2D to 2D "meet (touches)" topological relationship. Figure 7 depicts intersecting 2D surfaces that "meet (touches) at an interior of one surface.

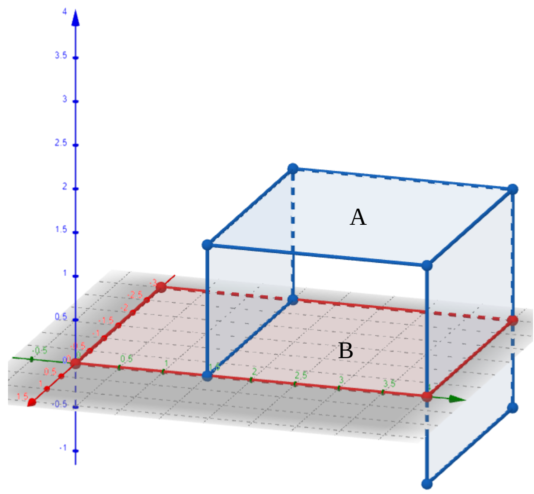

Figure 7. 2D to 2D "meet (touches)" topological relationship at an interior.

The 2D to 2D "meet (touches)" topological relationship where one surface intersects the interior of the other can be defined as follows;

Condition: 2D surfaces "meet (touches)" when the boundary of a surface intersects with the interior of the other surface.

The intersection matrix for the case in Figure 7 is expressed below;

Proof:

$$\dim(A,B) = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^- \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^o & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & \neg\emptyset \end{bmatrix}$$

where -1 = no intersection
0 = intersection at 0D point
1 = intersection at 1D line
2 = intersection at 2D surface
$\neg\emptyset$ = has intersection

Based on the intersection matrix above, the surfaces intersect at a 1D line boundary of Surface A and the interior of Surface B. The 1D line boundary of Surface B also intersects the interior of Surface B. No intersections occur between the interiors of both surfaces.

4.3 Preservation of Topological Information via Intersection Matrix

In 2D space, topological relationships consist of six topological groups, which are vertex-to-vertex, vertex-to-line, vertex-to-area, line-to-line, line-to-area and area-to-area. On the other hand, ten topological groups exist for objects in 3D space, which consist of vertex-to-vertex, vertex-to-line, vertex-to-area, vertex-to-volume, line-to-line, line-to-area, line-to-volume, area-to-area, area-to-volume and volume-to-volume. The intersection matrix describes the connectivity by examining the interactions between objects. In turn, the intersection matrix can be used to examine conditions set by topological rules that define valid topological interactions between objects to determine topological relationships. The use of 2D topological rules for objects in 3D space has been sufficient. However, it is required for 3D objects to be decomposed into lower-dimension objects to determine topological relationships based on 2D topological rules. The downside of this approach is the representation of topological relationships and connectivity is limited to 2D. Topological relationships between 3D objects are not able to be expressed as 3D interactions. 3D topological rules define valid topological interactions between objects in 3D space. In spatial databases, a DBMS is often assisted by spatial extensions to facilitate spatial data and spatial functions. Topological rules are implemented in the spatial extension, which facilitates the validation of geometries and spatial analysis. The topological rules are able to determine topological relationships based on the intersection matrices as well as return errors which describe invalid spatial objects.

5. CONCLUSION

The implementation of topology for building complexes adds value to 3D graphical output by defining valid topological interactions between 3D building units for 3D Cadastre. This will lead to topologically correct cadastral data and efficient spatial analysis in representing the RRR of the 3D building units. Enabling accurate representation of building units and the RRR will provide reliable information in facilitating building management, avoiding disputes, and supporting the planning of future urban developments. This paper attempted to define topological relationships that occur in 3D space. The conditions and proof based on the DE-9IM were presented for 3D to 3D objects and 2D to 2D objects. The "meet (touches)" and "overlaps" topological relationship were found to be crucial in 3D cadastre to determine adjacent or neighbouring 3D parcels. Meanwhile, topological relationships were also required to ensure valid internal connectivity within a 3D object.

In handling 3D objects, the topological properties that complement the geometries should also be accurately represented. Ideally, a 3D cadastre should be able to fully utilise 3D geometries as well as 3D topological properties. However, some challenges are currently present, such as complex data structures, voluminous data storage and higher demand for computation capabilities. Therefore, the fundamental definitions of 3D topological relationships specifically for 3D cadastre provide a practical subset of 3D topology.

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