OPTIMIZATION GEOMETRIC MODELS OF TRANSPORT NETWORK TRACING USED IN CITY PLANNING

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ABSTRACT:

The article discusses the improvement of methods of tracing transport networks in the context of optimal planning of cities and settlements. It is necessary to provide a systematic approach to planning. The development of the master plan considers the division of the territory into traditional functional zones. Functional zones are modeled in the form of circles and rectangles. Then these zones are divided into separate subsets. Objects located inside functional zones are modeled by points, and objects connecting these points by straight lines. Network tracing is achieved by constructing an orthogonal Steiner network taking into account the weight coefficients of the functional zones. Orthogonal or Euclidean Steiner networks are constructed in subsets. The construction of a network and pedestrian roads inside the functional zones is being carried out. When solving this problem, the calculation of the need for transport is also provided. Optimization of the transport network tracing is as follows: for a given set of points, it is required to determine the number and optimal location of additional points, so that the total length of the network is minimal. The shortest path to the points is determined. Optimization geometric models are an effective and visual means of developing various options for tracing the network between functional zones and within zones. From several network tracing options, a network that meets the pre-set planning requirements is selected. Allows you to analyze and make the right decision in determining the promising direction of development and construction of a city or settlement.

1. INTRODUCTION

Territorial planning at the level of a country, region, city or district is a key factor in sustainable social, territorial and economic development. It is located at the intersection of land use, the real estate sector and infrastructure development. In today's globalized world and in conditions of increasing urbanization, spatial planning, which used to be based on long-term forecasts, is subject to systemic disruptions or transformations under the influence of climate change, natural disasters, the globalized economy and the globalization of capital flows, as well as the growth of international migration. Territorial planning should promote the integration of sectors such as housing, transport, energy and industry, as well as the improvement of national and local systems of urban and rural development, taking into account environmental considerations (United Nations, 2020).

The main goal of modern cities, the meaning of life and doing business in cities is the breadth of opportunities for choosing jobs, goods and services, maximum labor and sales markets. Satisfaction with the life of each of us is determined, among other things, by the possibility of choosing the best workplace where we can reveal our abilities and get a decent reward; the best goods and services (including in the field of culture, healthcare, etc.) that satisfy our personal needs. Business efficiency is also largely determined by the choice of the best employees who will be satisfied with their work with moderate remuneration. This also applies to suppliers and customers. Transactions, purchases – as the main mechanism of economic development - are concluded as a result of meetings of people; the possibility of such meetings is determined by the mutual transport accessibility of people and organizations. The key mechanism for ensuring accessibility is transport. Due to the daily biological cycle of a person, the time of daily trips (the time that a person is willing to spend on moving to access the objects of his interest) in a normal situation does not exceed 3-4 hours a day, which is confirmed by numerous surveys and studies. Given that travel time is limited, it is possible to increase the number of available objects and people in only two ways – by increasing the density of accommodation (which we observe in cities) and increasing the speed of communication through the transport system (in order to cover as much territory adjacent to the place of residence and located on it during the time allotted for movement objects (United Nations, 2020). In the practice of designing, the choice of the configuration of the transport network is made taking into account the real conditions of the city in question in accordance with the structure of the economic and social complex and reflects the main directions of its development. Based on the above, we can conclude that the research is relevant.

2. PROBLEM STATEMENT

Improving the methods of tracing transport networks in the context of optimal planning of cities and settlements requires a systematic approach to design. With this approach, on the master plan, the considered territory of the city is divided into traditional functional zones. These zones can include residential, industrial, administrative, cultural, sports, as well as health and recreation facilities. The division of the territory under consideration into zones is necessary to create various options and choose the optimal configuration of the transport network tracing connecting these zones. Optimization of the transport network is as follows: for a certain set number of functional zones, it is required to determine the number and best location of additional points. At the same time, the total length of the network connecting these zones should be minimal. Next, optimal networks are built within each zone. At the final stage, in general, the shortest path of movement of vehicles to the desired points is determined. When tracing a network, an important indicator is the weight coefficients - interpreting capital and operating costs. Therefore,

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the weight coefficients show the significance of the functional zones under consideration. The efficiency and reliability of transport networks largely depends on the geometric parameter, in particular, on the length that affects its cost. It is known that the given costs are linear functions of transport networks. Therefore, when planning, first of all, the transport network of the smallest length should be considered. The solution can be reduced to the following geometric problem: a finite set of coplanar points is given and it is required to connect them with a line of the shortest length.

At the first stage of design, geometric models of transport networks are developed, then optimization problems are solved, which are reduced to various generalizations of the Ya problem. Steiner. The solution of the Steiner problem was investigated in (Kuspekov, Rotkov, 2016). The importance of the Steiner problem is confirmed by the fact that many scientists of the world conduct fundamental and applied research. In Cockayne (1989), variants of constructing networks in spaces with different metrics are proposed. In the planning process, functional zones are geometrically modeled in the form of coplanar rectangles and circles, the centers of the figures are dots, transport roads connecting these centers are lines. The solution of the problem is reduced to a generalization of Steiner's geometric problem of the shortest and minimum connection of coplanar figures and points with orthogonal and with Euclidean distances (Esmukhan, Kuspekov, 2012; Hanan, 1966; Kuspekov, 2022). The optimal configuration is achieved by constructing shortest and minimal Steiner trees connecting a given set of points with the introduction of additional points optimizing its solution.

3. SOLUTION METHODS

As a result of the study of the problem of tracing various types of transport networks, various configuration options have been developed and network properties have been identified. Methods for solving such and similar problems on the plane with Euclidean and orthogonal metrics are obtained. Based on the study and analysis of the proposed methods for optimal planning of the city's transport networks, a method of a systematic approach to the development of the city's master plan is proposed. This approach allows us to take into account the solution of many engineering tasks set out in the master plan. The essence of the technique is to develop optimization geometric tracing models and solution algorithms.

3.1 The first stage of planning

1 step. The territory of the city development is determined.
Step 2. In the master plan, the city is divided into traditional functional zones.
Step 3. Decisions are made on the number and configuration of the location of functional zones within the territory.
Step 4. When deciding on the configuration of zones, the strategy of the development of the city, transport networks and other engineering communications is taken into account.
Step 5. The outlines of the functional zones are geometrically modeled in the form of coplanar rectangles and circles.
Step 6. Rectangles and circles in the plan include residential, industrial, administrative, cultural, sports, health and recreation facilities.
Step 7. The significance of functional zones is determined and weight coefficients are introduced.
Step 8. Next, a mathematical method is used to divide a given set of points into separate subsets – in our case, a coplanar set of figures фигур $F = \{1,2,\ldots,f\}$ is visually divided into a local number of figures $G = \{G_1,G_2,\ldots,Gn\}$.

3.2 Second stage of planning

1 step. A certain number of families of subsets $G = \{G_1,G_2,\ldots,Gn\}$ from a finite set F are considered so that they form a partition of the set F so that the orthogonal distance between the sets of figures is minimal, the minimum is taken from the set of all possible partitions (Esmukhan, Kuspekov, 2012).
Step 2. The coordinates of the functional zones (rectangles and circles) are determined and the distances between them are calculated using the formula.

$$d(F_1,F_2)=|X_1-X_2| + |Y_1-Y_2| \quad (1)$$

where X_1,Y_1 are the Cartesian coordinates of zone F_1; X_2,Y_2 are the Cartesian coordinates of zone F_2.
Step 3. From the set of flat figures, two figures are selected, the distance between which is no greater than for any other two figures. The distance is calculated between the centers of the shapes. We connect the center points with a straight line segment, a minimum tree (MD) is formed.
Step 4. The next center point of the figure that joins the tree is determined.
Step 5. After constructing the MD, it may be necessary to connect two MD's at the next step and give rise to a new group of connected figures, a new minimal subtree is formed. Such subtrees should then be connected to each other based on the principle of the least elongation of the MD at each individual step of construction. To determine the coordinates of the additionally entered point and distance, we use various constructions of the Steiner tree (Esmukhan, Kuspekov, 2012).
Step 6. We obtain a single configuration of the shortest tree, the total length of all segments connecting the specified center and additionally entered points is minimal.
Step 7. Other network configuration options are being built.
Step 8. By the method of comparative analysis, the optimal minimum tree that meets the specified requirements in advance is selected from several network configuration options.
Step 9. At this step, we study objects located inside functional zones (rectangles and circles), identify the significance of these objects.
Step 10. We model objects in the form of points and enter weight coefficients.
Step 11. Based on the principle of least elongation, we connect the simulated points and build the shortest tree in each zone. We are building several configuration options for a network of transport and pedestrian roads. We select the final version of the network configuration.
For an orthogonal network, the distance between points is calculated by the formula (1). For the Euclidean model, the distance is determined by the formula

$$d(F_1F_2)=((X_1-X_2)^2+(Y_1-Y_2)^{2)\ 1/2} \quad (2)$$

Step 12. We adjust and build the final version of the network trace taking into account the main and secondary lines. After choosing the optimal configuration of the transport network, the types of vehicles are selected and the main parameters for the enlarged technical and economic calculation are determined.

4. IMPLEMENTATION OF TRANSPORT NETWORK TRACING

Consider the construction of geometric models of the transport network by the Steiner method on a plane with orthogonal and Euclidean metrics.

4.1 Orthogonal network tracing model

According to the first stage of planning, we will consider the construction of the main tracing lines connecting functional zones. Let on the plane given a set of functional zones F_1, F_2, F_3 F_f in the form of circles. We build an optimal network tracing configuration using the Steiner method. Building an orthogonal Steiner tree. To do this, you need to determine the number of Steiner points Nn, and their coordinates on the plane α. Building a minimum weight tree. The constructed minimal tree connects the vertices of the initial zones F_1, F_2, F_3 F_f. and the desired Steiner points $N_1, N_2, \ldots .N_n$.

Consider a finite set of eight functional zones zone F_1, F_2, F_3 F_4, F_5, F_6, F_7 and F_8. on the plane α is a rectangle of the building area. A positive value q_i, $i=1,2,$ f is associated with each Fi zone. The coefficient qi is called the weight of the Fi zone. It is required to build an optimal configuration of the transport network that meets economic and other requirements. To do this, the proposed method of the second stage of network trace planning is used
1 step. The location of the initial functional zones F_1, F_2, F_3 F_4, F_5, F_6, F_7 и F_8. is visually studied. and based on the properties of the shortest Steiner tree with an orthogonal metric, these zones are divided into three subsets, denoted by the numbers A, B, C., Figure 1. The distance between the starting points is calculated by the formula (1). Let the weight coefficients have the following relations: $q3 \geq q1+ q2+ q4$ and $q5 \geq q6+ q7+ q8$.
Step 2. Subsets A consists of the points of the center of the circles F_1,q_1; F_2,q_2; F_3,q_3 and F_8,q_8. Subsets B includes points F_4,q_4; F_5,q_5; F_6,q_6; F_7,q_7. Subsets C consists of points F_3,q_3 and F_5,q_5.
Step 3. Through the points of each subset A, B, C we draw lines parallel to the axes of the rectangular coordinate system. Thus, we define the boundaries of the subset A, B, C. The point F_5,q_5 is inside, and all the other starting points are located on the sides of the quadrilateral.
Step 4. Consider a subset of A. Calculate by formula (1) the distances between points F_1,q_1; F_2,q_2;F_3,q_3 and F_8,q_8 and F8,q8 and compare them. The distance between points A_3,q_3 and A_2,q_2 is minimal. Next, based on the principle of least elongation, we construct a minimal Steiner tree. Connect by orthogonal segments, a minimal Steiner tree MD_1 is formed for two points $F_3= F_2$, Figure 2. We build a quadrilateral with additional vertices O_5 and N_4.
Step 5. Compare the distances between MD_1 and F_1,q_1; F_8,q_8. We connect the segment F1, q_1 and MD_1. The Steiner point N_1 is determined and MD_2 is formed. Next, we connect MD_2 with the point F_8,q_8. There are two possible connection options, through the pivot point O_2, the Steiner point N_2 and the pivot point O_1. MD_3 is formed.
Step 6. Consider a subset B, Figure 1. The points of the zone F_4,q_4; F_6,q_6 and F_7,q_7 are located along the perimeter, and the point F_5,q_5 is inside the rectangle. Calculate and compare the distance between all the starting points. The distance between the points F_5,q_5 and F_4,q_4 is minimal. Connect by orthogonal segments, a minimal MD4 Steiner tree is formed for two points $F_5 = F_4$, Figure 2.
Step 7. Compare the distance between MD_4 , points F_4,q_4; F_6,q_6 and F_7,q_7. The nearest point is F_6,q_6. We connect it to MD_4 through the pivot point O_4. Next, we connect the point F_5, q_5 along two routes, through the turning point O_4 and the Steiner point N_4. A minimal MD_5 tree is formed.
Step 8. Combining the minimal MD_3 and MD_5 tree. To do this, we connect the Steiner points N_3 and N_4. As a result of the construction stages carried out, a minimal MD_6 tree is formed for the considered starting points.
Step 9. The minimal MD6 tree allows tracing of several network variants. Other MD6 structures are also possible for given points F_1, F_2, F_3 F_4, F_5, F_6, F_7 and F_8, within the mobility zone of rectangles. In our case, the final form of the trace configuration has the form shown in Figure 3. The minimal MD_7 Steiner tree, Figure 3, is the optimal orthogonal network connecting the original functional zones.

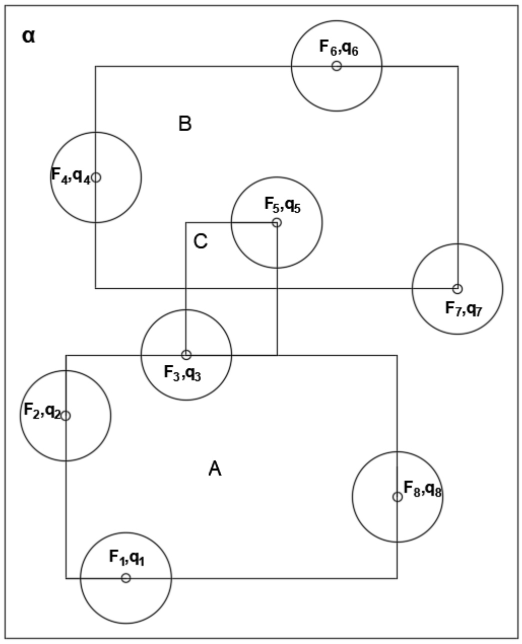

Figure 1. Splitting 8 source zones into three subsets A, B, C.

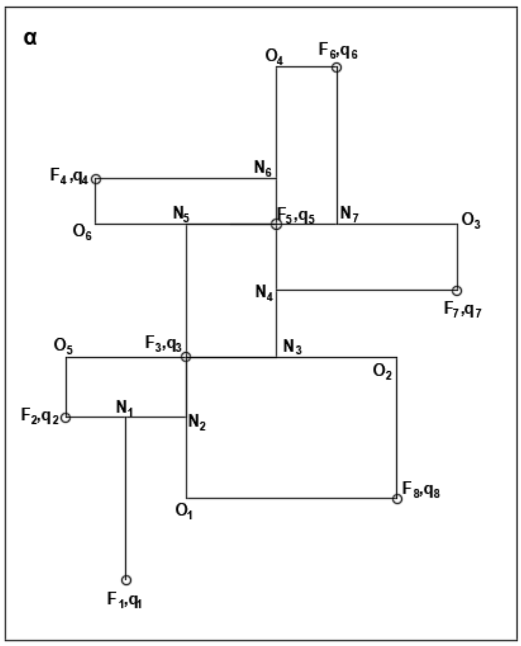

Figure 2. Stages of building a minimal tree Steiner.

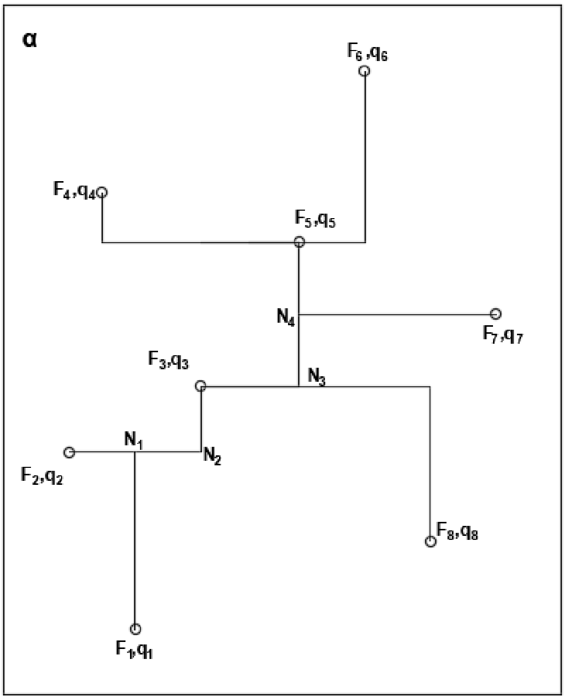

Figure 3. Optimal network tracing for eight functional zones.

4.2 Network tracing inside functional zones

Building an optimal network tracing configuration connecting functional zones allows you to identify the main and secondary roads.
Each functional zone consists of a different number of social and other types of objects. Therefore, the supply of engineering networks to these objects, including pedestrian and other ways of moving inside the zones, is also an important problem. Orthogonal and Euclidean network models can be used here. During the design process, all these objects are modeled by points, the elements connecting these objects, lines. This approach will allow us to apply the above methodology and algorithms of paragraph 4.1 to build a Euclidean network model.

4.2.1 Euclidean network model

Building an optimal network tracing configuration connecting functional zones allows you to identify the main and secondary roads. Each functional zone consists of a different number of social and other types of objects. Therefore, the supply of engineering networks to these objects, including pedestrian and other ways of moving inside the zones, is also an important problem. Orthogonal and Euclidean network models can be used here. During the design process, all these objects are modeled by points, elements connecting these objects, lines. This approach makes it possible to apply the above methodology and algorithms of paragraph 4.1 to build a Euclidean network model.

Let the functional zone consist of five objects. It is required to build a Euclidean network connecting these objects. We model objects with five points. To construct a minimal Steiner tree for various weight coefficients, we first construct KD for equal values of weight coefficients qi. In (Hanan, 1966; Kuspekov, 2010), the constructions of the minimal Steiner tree (MD) for three points are investigated and described, we use these constructions. Let three points M_1M_2 and M_3 with weights q_1, q_2 and q_3 be given, Figure 4.

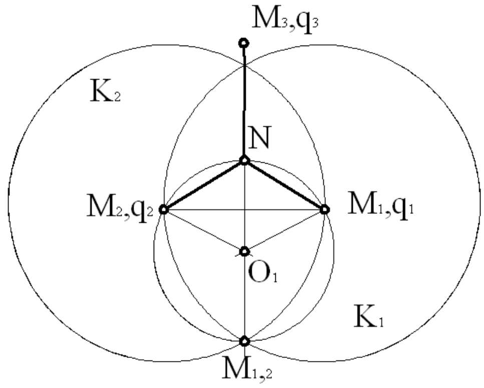

Figure 4. MD configuration for three points with equal values of q.

To determine the optimal configuration, the following constructions were carried out: circles with centers at points M_1 and M_2 were drawn the radius of the first circle K1 is equal, and the radius of the second circle K_2 is equal The points $M_{1,2}$ of the intersection of these circles K_1 and K_2 are equivalent to the points M_1 and M_2. Then an isosceles triangle $M_1M_2O_1$ was built on the side M_1, M_2. Then a circle K_3 is drawn and a straight line $M_{1,2}M_3$ which intersects intersect at point N, the desired Steiner point. The weight of the minimum tree (MD) for three points will be equal to the sum

$$\Sigma q_i = q_1|M_1N| + q_2|M_2N| + q_3|M_3N.. \tag{3}$$

In Kuspekov (2010), the necessary and sufficient conditions for the formation of a three-beam network are defined, where the properties of the vectors allow us to uniquely determine the scalar division and the gradient of the scalar field potential is written as follows:

$$grad = \frac{df}{dl_{oo}} = -q_1 + q_2 + q_3 \tag{4}$$

The extremum of the scalar field potential corresponding to the minimum of the function f will be at such a position of the point N when gradf = 0. Therefore, the minimum of the function f will occur at

$$q1+q2+q3 = 0 \tag{5}$$

that is, the vectors of the weights form a triangle. The values *q_1, q_2 and q_3* can be set as three segments of arbitrary length. In order for three arbitrary segments to form a triangle, they must satisfy the following inequalities:

$$q3 \geq q1 + q2; \quad q2 \geq q1 + q3; \quad q1 \geq q2 + q3 \tag{6}$$

The latter inequalities are necessary and sufficient conditions for the point N to be inside the triangle $M_1M_2M_3$. If one of the above inequalities does not hold, for example $q_1 \geq q_2+q_3$, then the point N coincides with one of the given points. We use these results to build a network. Consider the constructions for five points M_1,M_2,M_3,M_4,M_5, the configuration and select and select the optimal one that meets the specified conditions in advance. The given points M_1,M_2,M_3,M_4,M_5 form a convex pentagon. The equivalent points $M_{1,2}$, M_3,4 and $M_{(1,2)(3,4)}$ are determined by generalized Steiner constructions for three points, Figure 5. A circle drawn through $M_{1,2}$ $M_{(1,2),(3,4)}$ and $M_{3,4}$ intersects the straight line M_5 $M_{(1,2)(3,4)}$ at point N_1, Figure 5.

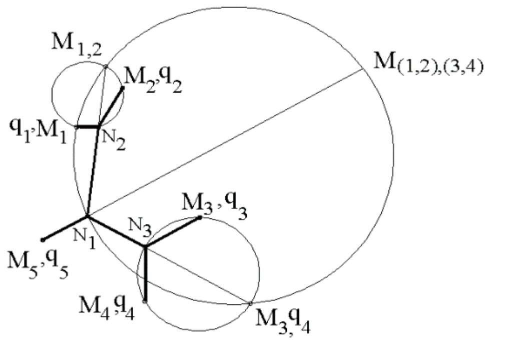

Figure 5. MD configuration for five points with equal values of q.

A circle drawn through the points M_1, M_2 and $M_{1,2}$ intersects the line N_1 $M_{1,2}$ and the position of the point N_2 is determined. The straight line $N_1M_{3,4}$ intersects the circle drawn through the points M_3,M_4,$M_{3,4}$ at the Steiner point N_3. The total minimum length of the MD_5 tree is

$$L=q\cdot(|M_1N_2|+|M_2N_2|+N_2N_1|+M_3N_3+M_4N_3|+N_1N_3|) \qquad (7)$$

Consider the configuration at different weight values. Let $q_1 \geq q_2+q_3+q_4+q_5$. Based on the principle of least elongation, the network configuration shown in Figure 5 is transformed, and a new configuration of the minimum tree is obtained, Figure 6, with $N_2 = M_1,q_1$.

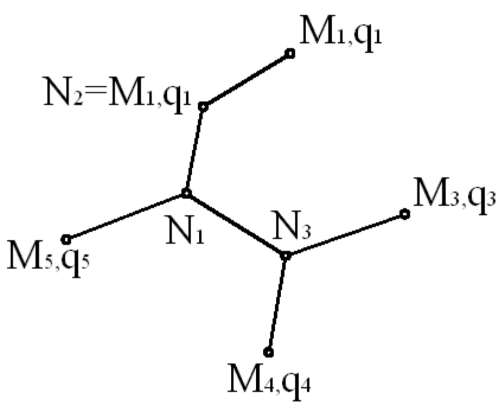

Figure 6. MD configuration for five points at q1≥q2+q3+q4+q5.

After building a network trace for functional zones and determining the main, secondary lines, options for connecting the line to the internal objects of the zones are considered.

4.2.2 Orthogonal network model

Let be given in the plane a set of points M_1, M_2, Mn, in each of which some positive quantity q_1, i= 1,2,....m, q_1 is the weight of the point. It is required to determine the number n, place a minimum weight tree on the plane with vertices at points M_1, M_2,. Mn, N_1,N_2,...Nn. The value of qi|MiNi| is called the weight of the MiNi branch, and the weight of the tree is called the sum of all the branches of this tree. A tree satisfying the conditions of the formulated problem is called minimal (MD). The qi weighting factors are interpreted as specific capital and operating costs.
The task is to design engineering networks with a minimum of costs for its construction and operation.
Consider the MD configuration for three points of the plane M_1,q_1; M_2,q_2 and M_3,q_3 with weighting coefficients q_1=q_2=q_3, Figure 7, (Esmukhan, Kuspekov, 2012).
We draw straight lines parallel to the axes OH and OH through the given points, we get a three-beam connection and a Steiner nodal point that optimizes the solution of the problem.

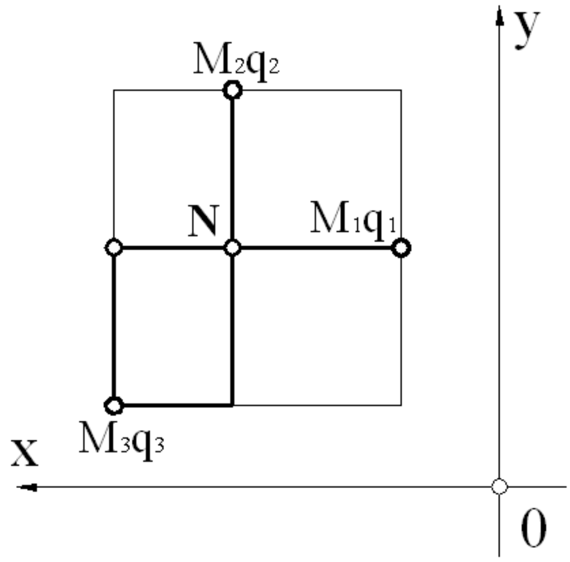

Figure 7. A three-beam tree at q1=q2=q3.

We draw straight lines parallel to the axes OX and OY through the given points, we get a three-beam connection and a Steiner nodal point that optimizes the solution of the problem.
The total length of MD_3 has the following expression: L=q_1|M_1|+ q_2|M_2| + q_3|M_3N|. If $q_1 \geq q_2+q_3$, then the point N coincides with the point M_1, figure 8. If $q_2 \geq q_1+q_3$ and $q_3 \geq q_1+q_2$, then N=M_2 and N=M_3, other MD_3 configurations are obtained. Comparing the total length L for all four configurations, it is revealed that the minimal tree is constructed in the case q_1=q_2=q_3.

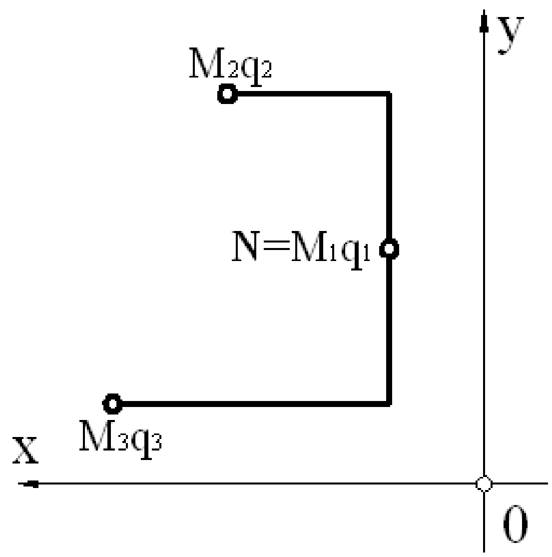

Figure 8. Trunk tree at $q_1 \geq q_2+$.

Consider constructing the shortest connecting line for five points of the plane. Let the points M_1, M_2, M_3, M_4 and M_5 be given with weights q_1, q_2, q_3, q_4 and q_5 , Figure 9. Based on the principle of least elongation, we construct the following MD_5 configuration, let $q_1=q_2=q_3=q_4=q_5$.
Step 1. Calculate the distances between the given points using formula (1).
Step 2. Select a pair of points having the shortest distance. Such a pair is the points M_1 and M_2, which can be connected to each other by the shortest line $M_1 N_1 M_2$ or $M_1 N_2 M_2$, figure 9.
Step 3. Connect the points M_3 and M_4 with a polyline $M_3N_3M_4$ or $M_3N_4M_4$. We get the fragment $M_3 = M_4$.
Step 4. Compare the distances between the fragments $M_1=M_2$, $M_3=M_4$ and the point M_5. The nearest neighbors were the fragment $M_3 = M_4$ and the point M5, which is located in the trunk zone. Therefore, $M_3=M_4$ is connected to M_5 by the nearest vertex N_3 through the pivot point N_5 or N_6. After that, point N_3 becomes a Steiner point and the mobility zone of the network changes.
Step 5. Combine the two fragments obtained in step 2 and 4.
Step 6. After merging, we get the final configuration built for the minimum MD_5 tree, Figure 10. For this configuration, other equivalent connection options for the desired points are possible. The shaded part is the mobility zone of the network for two points. We get the fragment $M_1 = M_2$.

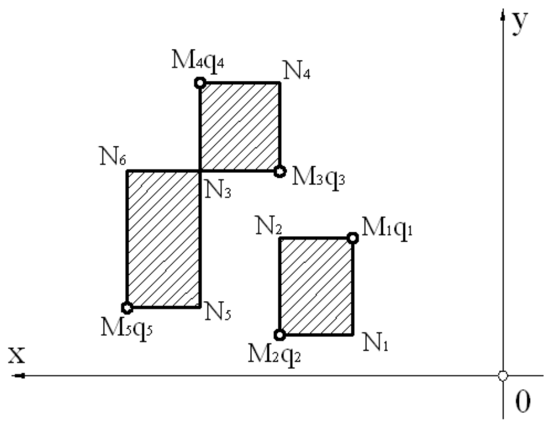

Figure 9. Minimum tree for five points.

At different values of q, we have obtained other different options for tracing the connection of MD_5 points.

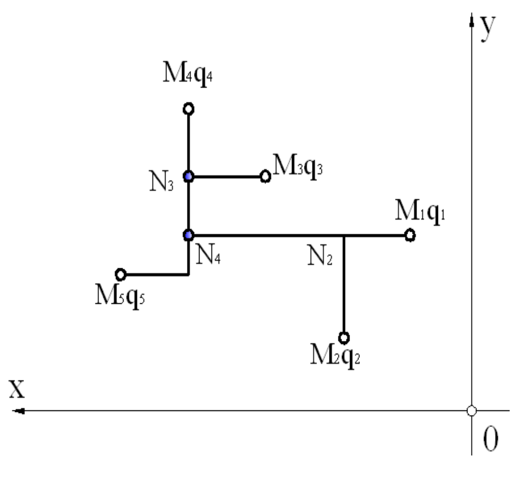

Figure 10. Tracing for five points.

Thus, the constructed configurations of minimal trees by the method of least elongation allow us to obtain several variants of minimal trees of the same length, determined by the mobility zone. This circumstance makes it possible to take into account in the actual design of engineering networks for various purposes, along with the length of connecting lines, other factors affecting the total cost of the network.

5. CONCLUSIONS

Planning and research of the properties of the transport network and the construction of its optimal configuration, satisfying pre-set conditions, is a complex and multivariate engineering and economic task. Therefore, the development of a network geometry or configuration that reflects the overall planning structure of the city makes it possible to improve the city development strategy Geometric modeling of the tracing of transport networks is considered from the position of the Euclidean plane and the construction of the shortest connecting lines for a given set of points, taking into account the weight. For this purpose, some methods of determining Steiner points and constructing shortest connecting lines, constructing a minimal tree with orthogonal and Euclidean metrics have been studied, systematized and generalized, satisfying some pre-set requirements. The advantage of this method is that after each step of construction, a network configuration is formed, if it optimally meets the pre-set conditions, you need to stop and adjust the network. Designers have the opportunity to analyze other problems, for example, increasing the capacity of streets, building roads and their improvement, optimizing the route network, organizing parking lots and other facilities, coordinating the activities of various organizations, managing and controlling the operation of transport, traffic flow management, improving the quality of public services and passenger transportation safety, traffic safety issues. It helps to effectively form individual links of the transport network that specialize in passing certain types of flows, such as bus or bicycle, which improves the quality of transport services and leads to the creation of certain structural properties of the network. The development of a package of application programs based on the methodology and algorithms allows you to calculate all possible options for building a network.

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