Geometric Models of Local Optimization of Highway Networks that Improve the Configuration of Tracing in Functional Zones

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Abstract

The article discusses the improvement of methods for discrete local optimization of highway tracing in the context of planning urban transport networks. Tracing and building the optimal configuration of the highway network is one of the key tasks in city planning. The main goal is to determine the shortest route for moving vehicles. Delivery of people to their destination and goods to consumers in a short time. The discretely local optimization of the network for three given points is considered. Network tracing for these points is achieved by building a polar Steiner network. Along with the orthogonal and Euclidean distance, as the research results have shown, the polar distance is important in practice. To introduce the polar distance, let us consider a certain plane with a fixed polar coordinate system. For some practical reasons, on a plane with a polar coordinate system, we leave only two directions of movement free. Movements are allowed along concentric circles drawn from the center coinciding with the pole, and along rays emanating from the pole. A bundle of straight lines with a support at the pole and the set of all concentric circles drawn from the center form an orthogonal polar grid. To solve this problem, various variants of geometric network models with a polar metric for three points are systematized and generalized, taking into account the weight of the specified points. The synthesis of an optimal highway route connecting specified points is a necessary component of optimizing the city's transport networks. To solve the problem, a network configuration with a polar metric consisting of radial segments and circular arcs is considered. The total length of the segment of arcs and circles should be minimal. The optimal network configuration is achieved by adding an additional Steiner point. The network constructed in the polar coordinate system will be called the "Steiner Polar Network". Geometric models of local optimization are an effective and visual means of developing various network tracing options within functional zones. From several network tracing options, a network is selected that meets the pre-defined planning requirements. It allows you to analyze and make the right decision in determining the promising directions for the development of the city's transport network.

1.Introduction

Tracing transport networks in urban planning, taking into account functional zones, is a key design task (Kuspekov K.A., 2023). The main goal is to determine the shortest route for moving vehicles. Delivery of people to their destination and goods to consumers in a short time. Transportation is one of the main components of the modern market environment. In addition, transport occupies an important place in the development of the company, so any company operating in the market is in close cooperation with the external environment and largely depends on it. The manufacturer receives all the necessary materials and components from suppliers, then delivers the finished products to the consumer through intermediaries. It is thanks to this interaction that objects are moved. In this case, companies are interested in improving the quality and speed of goods transportation. Transportation logistics addresses these issues by building optimal cargo delivery routes (Nerush, 2019). The problems of building vehicle routes to avoid high-risk areas or statistically significant metaclusters of road accidents, that is, clusters resulting from secondary clustering of clusters of road accidents that occurred in Springfield in 2013-2018, have been investigated and shows the importance of optimal road network routing (Gerstein, 2022). The synthesis of an optimal highway route connecting the start and end points is a necessary component of optimizing highway networks. Towards modern approaches in the field of tracing road networks the probabilistic road map method (PRM) is applied (Geraerts, 2002; Kavraki,1994; Topcon, 2022).

In the context of analyzing and solving the above-mentioned tasks, it is relevant to improve methods for discretely local optimization of tracing highway distribution networks by using optimization geometric models to build roads bypassing obstacles of minimal length that meet design requirements. The time of arrival at the destination is selected as the route selection criterion. In design practice, the choice of the transport network configuration is made taking into account the real conditions of the city in question in accordance with the structure of the economic and social complex and reflects the main directions of its development (Kuspekov K.A., 2023). Based on the above analysis, it can be concluded that the research objectives are relevant.

2. Setting the Task

In the context of planning the city's transport networks, the tasks of local optimization of highway tracing in functional areas arise. As is known, such zones include residential, industrial, administrative, cultural, sports, as well as health and recreation facilities (Kuspekov K.A., 2023). Improving the configuration of highway tracing is one of the most important design stages. Since during the design process, various conditions of restrictions may arise on the route path. There are problems with laying a route bypassing obstacles, and many areas of other routes have to be identified. The complexity of solving engineering tracing problems is due to the fact that these problems belong to extreme and NP-hard discrete optimization problems. The main goal of local network optimization is to build rational routes for delivering people to their destination and goods to consumers in a short time. This local optimization task of highway tracing should be based on the following requirements:

1) The network should cover all specified points, without violating restrictions on the way.

2) The optimal geometric parameters of the network should have a total minimum length, which reduces the time for transport to travel the section of the path in question.

3) Other dimensions such as the width and curvature of the road that affect traffic safety must comply with the requirements of the state standard and building design standards.

4) To achieve the lowest possible construction cost of roads on the considered section of the highway.

5) The prospect of improving the network configuration by avoiding obstacles and with optimal access to the main roads without creating congestion on the roads.

The list of requirements for the projected network and the analysis of the conducted scientific research shows that an important stage in calculating the tracing of highway networks is its chosen configuration. The article discusses the improvement of methods for discretely local optimization of tracing highway distribution networks using the construction of geometric models. For this purpose, various variants of geometric network models for three points are being developed and applied. The main criterion for optimality is the travel time, cost of construction and operation of roads. This cost depends on the length of the road and on the passage of sections of the highway with restrictions on the way. It is necessary to take into account possible obstacles on the shortest route of the transport route. Obstacles can be in the form of built-up areas, forest plantations and other important objects of the master plan. The synthesis of an optimal highway route connecting specified points is a necessary component of optimizing highway networks. To solve the problem, curved routes along the arc of a circle are considered, taking into account obstacle avoidance. The path consists of straight lines and arcs of circles._If the size of the objects is small compared to the length of the structures connecting them, then the geometric model of the engineering network consists of a finite set of points to be connected by a line of the minimum (shortest) length. If the dimensions of the engineering facilities are commensurate with the length of the structures connecting them, then the geometric model of the engineering network consists of a finite set of geometric shapes to be connected by a line that meets the pre-defined requirements. Obviously, it is possible to construct countless lines connecting given points. Any line connecting the specified points will be called a valid line. The task is to find a line from among the acceptable lines that meets the requirements. It is necessary to distinguish between necessary and sufficient conditions under which an acceptable line will meet specific requirements. Necessary and sufficient conditions can be specified after establishing the criterion of optimality of the connecting lines. Obtaining necessary and sufficient conditions is the main task in optimization theory. This task is very difficult. For example, consider the problem of the shortest connection of points in a plane. As a result of many years of research, scientists have established only the necessary conditions that the shortest line must meet. As for the sufficient conditions for the shortest lines, they have not been obtained so far. Therefore, this task is still not solved. After establishing the necessary and sufficient conditions, another task arises, which boils down to defining algorithms for constructing connecting lines that satisfy the necessary and sufficient optimality conditions. An acceptable line satisfying the necessary conditions will be called a relatively (locally) optimal line. Various configurations of the geometric network model for two points on a plane with a polar metric are systematized and generalized. Various configurations of the geometric network model for two points on a plane with a polar metric are systematized and generalized. A network configuration with a polar metric is being built. The local optimization of highway tracing is as follows: for a given set of three points, it is necessary to determine the number and optimal location of additional points, so that the total length of the network is minimal. For engineers designing highways, optimization geometric models are an effective and visual means of developing various options for local network optimization while bypassing obstacles on the route. You can build several network tracing options. The comparative method is used to select a network that meets the specified design requirements in advance. The constructed double-radial segments practically reflect the real projection of highways.

3. Solution Methods

The results of the study show that optimization and generalization of various configurations of the highway network structure are considered in the work (Nemchinov, 2016). This article offers a variety of schemes in the planning of tracing in the construction of highways - radial, radial-ring, rectangular, rectangular-diagonal, triangular, combined, free in the form of radial and linear. Geometric models of transport networks in city planning for a given set of points with orthogonal and Euclidean metrics have been investigated and a methodology described (Kuspekov, 2023). In the practice of designing highway networks, along with Euclidean and orthogonal metrics, the use of polar distances is important. When designing and discretely optimizing the tracing of highway distribution networks, it is also important to build a curved highway. Curved paths can be approximated by straight lines and arcs of circles. The geometric model of such optimization problems is a plane with a polar metric (Esmukhanov, 2011). Based on the study and analysis of the proposed methods for optimal planning of the city's transport networks, a methodology is proposed for developing an improved configuration of the highway network in functional areas. The methodology should take into account the prospects for improving the network configuration by avoiding obstacles and with optimal access to the main highways of the city. The essence of the technique is to develop optimization geometric tracing models and algorithms for solving on a plane with a polar metric. Let's assume that three fixed points M1,q1; M2,q2; M3 ,q3 are set on the plane from the polar coordinate system. It is required to connect the three points with arc segments of a circle and radial segments so that the total length is minimal. In this formulation of the problem, it is advisable to apply a configuration constructed for three points (Kuspecov, 2009). The optimal network configuration is achieved by adding an additional Steiner point, similar to solving transport problems with orthogonal and Euclidean metrics. The network constructed by the polar coordinate system is called the "Steiner Polar Network" by analogy with the "Euclidean Steiner network" and the "Orthogonal Steiner network". The Steiner point allows you to build a minimum-length network and various options for connecting fixed points in the network's mobility area. The points are connected by radial and arc segments of a circle. Using the method of comparative evaluation, we can formulate and use an algorithm for constructing a tree of the shortest length passing through a given set of coplanar points (Kuspekov, 2023):

- Two points Mi and Mj are selected, the distance between which is smaller than for any other pair. KDSH2 (the shortest Steiner tree) is being constructed.
- Each subsequent step of the algorithm consists in the transition from KShT, constructed for a group of points from t points, to KShT+1 for a group of t+1 points. In this case, the following are defined: a) the next t+1 point, which should be connected to the tree; b) the configuration of the CDSHT+1, to which the previously found CDSHT will enter, in the general case, already partially deformed.
- After constructing the CDT, it may be necessary to connect two points close to each other at the next stage, which are not included in the CDT and give rise to a new group of connected points, i.e. a new shortest subtree is formed. Such subtrees should be combined with each other in the prescribed order, based on the principle of the smallest elongation of the CDT at each individual stage of its construction.
-The function (1) below is determined by the location of points M_1 and M_2 on a plane with a fixed polar coordinate system:

$$d(M_1M_2) = \begin{cases} \rho_1 + \rho_2, if\ |\varphi_1 - \varphi_2| \geq 2 \\ |\rho_1 - \rho_2| + |\rho_1(\varphi_1 - \varphi_2)|, if\ |\varphi_1 - \varphi_2| < 2u, \rho_1 < \rho_2, \\ |\rho_1 - \rho_2| + |\rho_2(\varphi_1 - \varphi_2)|, if\ |\varphi_1 - \varphi_2| < 2u, \rho_1 > \rho_2 \end{cases} \quad (1)$$

This approach is one of the rational ways to build an optimal network tracing configuration connecting specified points with circular arcs bypassing obstacles, and also allows you to take into account other criteria for the curvature of the route.

4. Results and Discussions

To solve this problem, we choose a plane with a polar coordinate system and fixed points with known coordinates. The conducted studies (Kuspekov, 2009; Esmukhan, 2012) allow us to construct various geometric network models with a polar metric for local points, the constructed configurations reflect images of real highway networks. Using the considered polar coordinate system, we allow and leave only two directions of movement free. Movements are allowed along concentric circles drawn from the center coinciding with the pole, and along rays emanating from pole 0. A bundle of straight lines with a carrier at point 0 and the set of all concentric circles drawn from the center O form an orthogonal polar grid. For clarity, let's consider constructing the shortest path between two points M_1 and M_2 (Figure 1). From point M_1 to point M_2, you can walk along the arcs of concentric circles drawn from pole 0 and along straight lines passing through pole 0. The set of all possible lines connecting M_1 and M_2 forms a curved quadrilateral M1NM2N1. The two sides M1N1 and NM2 form arcs of concentric circles, and the other two sides M1N and N_1M_2 form straight line segments. In the considered mobility zone of the curved quadrilateral network, various paths can be constructed connecting the points M_1 and M_2, consisting of dougo-radial segments. Obviously, the shortest line connecting the points M_1 and M_2 consists of a straight line segment M_1N and an arc NM_2 of a circle, highlighted in Figure 1 by a thickened line.

It is known that any function depending on the location of points M_1 and M_2 can be considered as the distance between them if this function is always positive and symmetric, and there is also the so-called triangle inequality, in addition, this function must be non-degenerate. Indeed, this function has the specified distance properties.:
1) $d(M_1, M_2) \geq 0$, i.e. the function is always positive;
2) $d(M_1, M_2) = d(M_2, M_1)$ (symmetry); the distance from point M_1 to point M_2 is equal to the distance from point M_2 to point M_1;
3) the distance from point M_1 to point M_2 is equal to or less than the sum of the distances between points M_1M_3 and between points M_3, M_2,i.e. $d(M_1,M_2) \leq d(M_1,M_3) + d(M_3,M_2)$ (triangle inequality);
4) $d(M_1, M_2) = 0$ if and only if the points M_1 and M_2 coincide (non-degeneracy).

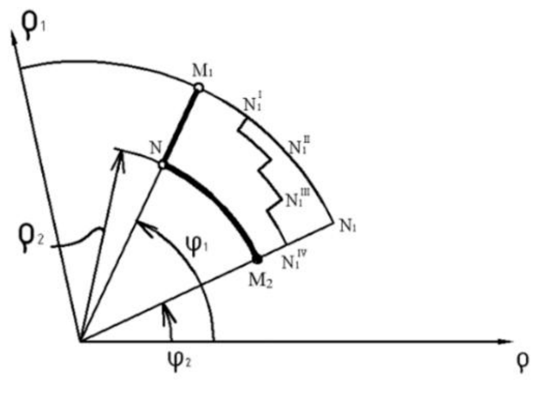

Figure 1. Network mobility zone in a curved the M1NM2N1 quadrilateral.

On a plane with a polar metric, unlike a plane with a Euclidean metric (depending on the distance of points M_1 and M_2 relative to the selected polar coordinate system), there are segments of various types connecting the specified points with the shortest line (Esmukhan and Kuspekov, 2012). Definition: The "segment" connecting the points M_1 and M_2 of the E_2 plane with the polar metric is called such a set of points E M_1, M_2 of this E_2 plane that for any point M_3 of the set EM_1, M_2 the equality holds:
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$$d(M_1,M_2) = d(M_1,M_3)+d(M_3,M_2) \quad (2)$$

For different points $M_1p_1\ \varphi_1$) and M4 (p2, φ_2), depending on their location, the following special cases are possible.

1. Let $\varphi_1 = \varphi_2$, $p_1 \neq p_2$. Then the distance between points M_1 and M_2 based on the above function will be equal to $d(M_1,M_2) = |p_1 - p_2|$, and the set of points E M_1,M_2 for which equality (2) holds fills the gap between points M_1 and M2 (Fig.2), while the points M_1,M_2 and 0 are collinear.

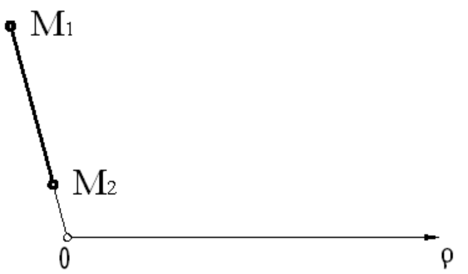

Figure 2. Radial connection of two points with a polar metric.

In this case, the concepts of a "segment" on planes with polar and Euclidean distances coincide.

2. $\rho_1 = \rho_2$, $\varphi_1 \neq \varphi_2$, $|\varphi_1 - \varphi_2| < 2$. Then the distance between the points M_1 and M_2 will be equal to $d(M_1,M_2) = \rho\ |_1 (\varphi_1 - \varphi_2) = |\rho_2(\varphi_1 - \varphi_2)|$, and the set of points E M_1, M_2 fills the gap between the points M_1 and M_2 along the arc of a circle centered at point 0 and with a radius equal to (Figure 3).

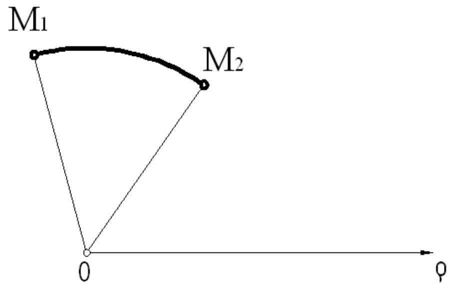

Figure 3. Arc connection of two points with polar distances.

3. If the distance between points M_1 and M_2 based on the above function is equal to $d(M_1,M_2)=p_1-p_2$.The set of points E M_1, M_2 consists of two segments $[M_10]$ and $[0M_2]$, the angle between which is at least 115° (Figure 4).

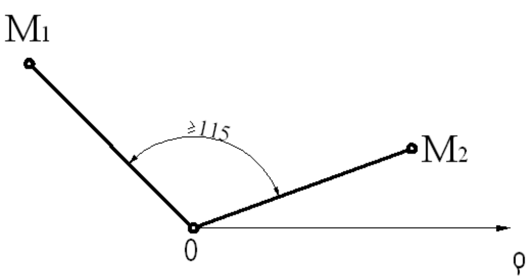

Figure 4. Backbone segment for two points with polar distances.

4. Let $p_1 \neq p_2$, $\varphi_1 \neq \varphi_2$ and $|\varphi_1 - \varphi_2| < 2$. Then the distance between points M_1 and M_2 (assume that $p_2 < p_1$) will be equal to $d(M_1,M_2) = |p_1 - p_2| + |p_2(\varphi_1 - \varphi_2)|$. and the set of points E M_1, M_2 fills the polyline M_1NM_2, consisting of a segment M1N and an arc NM_2 (Figure 5). This general case can be divided into two cases, one of which coincides with the first, and the other with the second, the special cases discussed above.

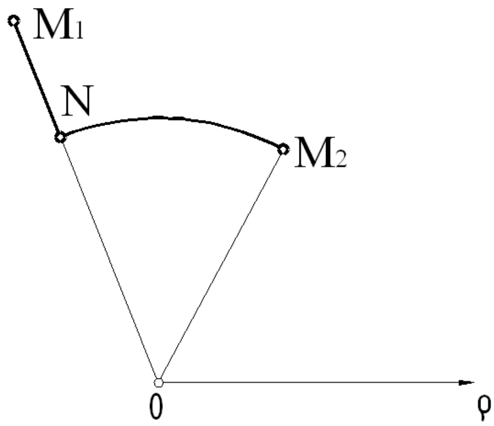

Figure 5. Dugo-radial connection of two points.

Next, we consider optimization geometric models of a minimal tree for three points: and with weights and (Kuspeckov, 2009). Let , , and with different configurations be set on a plane relative to a fixed polar coordinate system. Configuration 1 is colinear-radial: the points are located on the same radial straight line (Figure 6). In this case, a

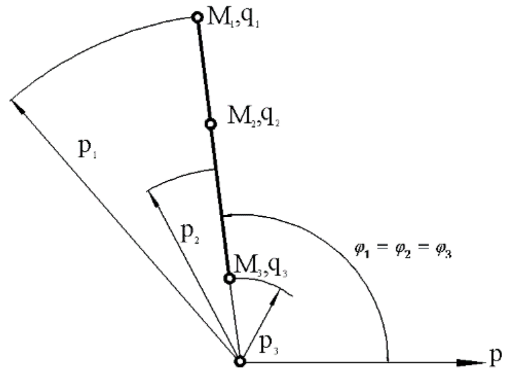

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Figure 6. Radial configuration.

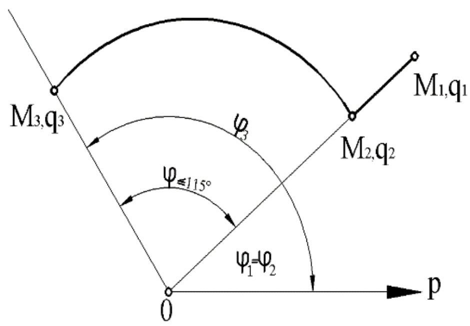

Figure 7. Double – radial configuration.

Then the length of the minimal tree will be equal to, the length of the shortest connecting point and , the line i.e. . If , then the point . If , then , if it turns out , then . , i.e. , the minimal tree consists of linear radial segments and topology 1 is optimal for the case . Configuration 2 is arc–radial: let the points and with weights be arranged as shown in Figure 7,

$$\rho_1 \neq \rho_2 = \rho_3, \varphi_1 = \varphi_2 \ .$$

Let's apply the method of comparative estimation, the shortest state is the radial segment connecting the points , and in the second step we connect the points with an arc segment if . The length of the tree , at and . Then , at and . Then. The resulting tree consists of arcs about radial segments. Suppose , then , if , then , i.e. topology 2 is appropriate in the case . Configuration 3 is the backbone minimum tree when, (Figure 8).

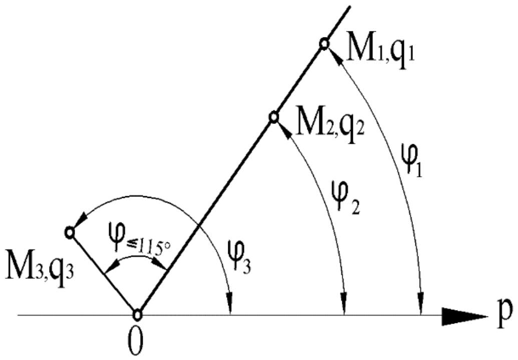

Figure 8. Backbone configuration.

In such an arrangement of points , and . Length Let's consider when points with weights and occupy a more general position on a plane with a polar metric. Configuration (Figure 9) the points and are located radially and . By the method of comparative evaluation, we determine that at the first step the shortest segment is formed by a radial fragment. In the second step, we connect the point to the fragment along the arc of a circle from the point , i.e. a nodal point with coordinates is formed that minimizes the total length.

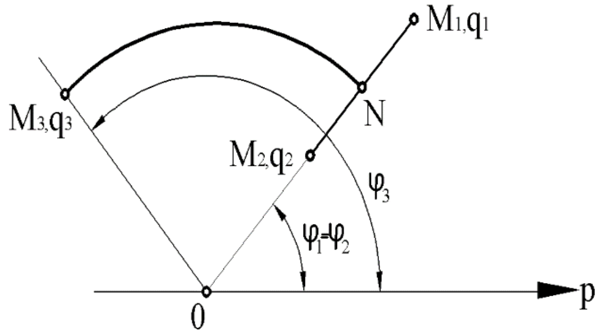

Figure 9. Configuration of a minimal tree with a nodal point.

Let , then , if , then , at , . We see that with different values, the configuration of the minimal tree also changes. Configuration 5: All points occupy the position shown in Figure 10. They are located inside the zone, at an angle. There are two possible connection cases here:
1) we connect the points ,
2) the point is a straight line segment, then c connects to the arc of a circle drawn from the center of O; after that, we draw a straight line that intersects the arc at the nodal point.
Comparing the lengths in the two cases, we determine that in the second case, the given points and are connected by a shorter line through the nodal point . From this case, we establish the following: if we consider it as an isosceles triangle with an angle at the vertex, then inside such a triangle there is a nodal point connecting the points and a line of the shortest length.

Comparing the distance between the points in the first step, we connect the points and , as located close to each other through the nodal point . In the second step, we connect the dot to the dot through the dot. Coordinates , because and After building a network trace for functional zones and determining the main and secondary lines, options for connecting the line to the inner objects of the zones are considered.

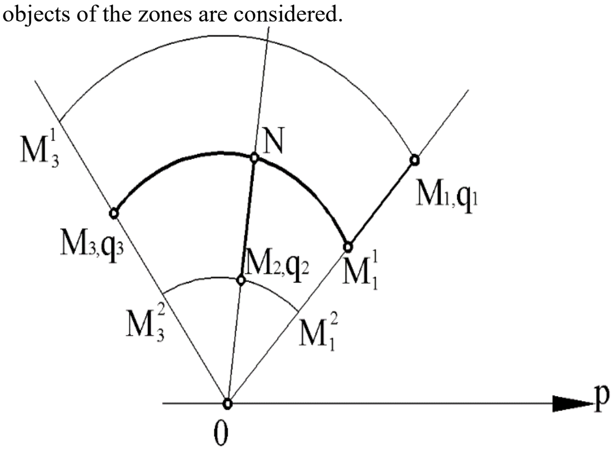

Figure 10. Configuration of a minimal tree with a transition point.

4.1 Algorithm for Building a Lan Tracing for Four Points

Step 1. The territory of tracing the local highway network in the city development plan is determined.
Step 2. The coordinates of the desired points to be connected by the shortest lines are determined.
Step 3. The construction area of the route between the points is being studied for restrictions by obstacles of various kinds.
Step 4. Consider a plane with a fixed polar coordinate system, with a given pole and a polar axis. A bundle of straight lines with a support at the pole and the set of all concentric circles drawn from the center O form an orthogonal polar grid.
Step 5. Let's draw the dugo-radial segments through the desired points and construct a curved quadrilateral.
Step 6. We apply heuristic methods for constructing a route, compare the distance between points, and solve the Steiner problem on a plane with a polar metric.:
- Two points are selected from the set of points, the distance between which is not greater than for any other two points. The distance between the points is calculated . We connect the points with radial or arc segments, forming the shortest tree.
-The next point that joins the tree is determined.
-After constructing the shortest tree, it may be necessary to connect two shortest trees at the next step and give rise to a new group of connected shapes, i.e. a new shortest subtree is formed. Such subtrees should then be connected to each other based on the principle of least elongation of the shortest tree at each individual step of construction. To determine the coordinates of the additionally entered point and the distance, we use various Steiner constructions (Esmukhan, 2012).
-We get the configuration of the shortest tree. The total length of all the dougo-radial segments connecting the specified and additionally entered points is minimal.
-Other configuration options for the polar network for a given mobility zone are being built. -A comparative analysis selects the optimal LAN configuration that meets the design requirements.
Step 7. We are correcting and building the final version of the network trace configuration. We make a technical and economic calculation.

5. Conclusions

Designing, researching the properties of the highway network and building its optimal configuration to meet the pre-set conditions is a complex and multivariate engineering and economic task. In this regard, the use of geometric network models or configurations that reflect the overall planning structure of the territories in question in terms of the topographic surface of the city remains relevant. Optimization geometric models make it possible to analyze and improve the development strategy of highway tracing. The discrete-local optimization of the highway network is considered from the position of the plane by the polar metric and the construction of the shortest connecting lines for a given four points. Some methods for determining the Steiner point and constructing the shortest connecting lines on a plane with a polar metric are systematized and generalized. Using the comparative method, the shortest tree with a polar metric is constructed that satisfies some predefined requirements. Several options for the shortest route are being built to avoid obstacles on the route. The advantage of geometric models and the proposed methods of problem solving is that after each step of construction, the network configuration is formed, if it is optimal and meets the pre-set conditions, you need to stop and adjust the network. Engineers have the opportunity to analyze and solve other problems, for example, increasing the capacity of the road, one-way and two-way, circular traffic, landscaping in the area of network mobility. This design approach makes it possible to improve the quality of public services and the safety of passenger transportation, as well as issues related to the organization of traffic safety. Strategic helps to effectively form the necessary links of other types of transport network, which leads to the creation of certain structural properties of the network.

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