Inside Bramante: Best Fit Algorithms for Architectural Analysis

Carlo Bianchini [1], Flavio Carnevale [2], Marika Griffo [3]

[1] Dept. of History, Representation and Restoration of Architecture, Sapienza, University of Rome, Italy – carlo.bianchini@uniroma1.it
[2] Dept. of antiquities, Sapienza, University of Rome, Italy – flavio.carnevale@uniroma1.it
[3] Dept. of History, Representation and Restoration of Architecture, Sapienza, University of Rome, Italy – marika.griffo@uniroma1.it

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Abstract

The use of massive digital surveying is now a well-established practice for the documentation and monitoring of cultural heritage. Our research focuses on point cloud analysis through developing best fit algorithms to expand the toolkit available for critical historical studies. By merging advanced computational methods with traditional analytical approaches, the study aims to support the interpretations process of heritage sites and artifacts, enriching scholarly research. The integration of computational techniques with traditional methods will enable new forms of analysis, enhancing the knowledge, preservation, and valorization of cultural heritage. The research focuses on three Renaissance structures designed by Donato Bramante-The Santa Maria della Pace cloister, the Vatican staircase and the Tempietto of San Pietro in Montorio- and tests best-fit algorithms on column shafts to detect geometric patterns and possible anomalies.

1. Introduction

The study of historical architecture inherently combines both qualitative and quantitative aspects. However, traditional research in the humanities has often emphasized indirect sources—such as textual and graphic documentation—over direct analysis of the built artifact. This paper proposes a complementary approach, applying best-fit algorithms to 3D point clouds in order to uncover hidden geometric and formal features. By examining architectural structures through computational methods, we aim to enrich established methodologies with new forms of evidence, rooted in direct observation and measurement.
This perspective is grounded in the idea that each monument is, first and foremost, the primary source of its own history (Bianchini et Al., 2024). Through a data-driven approach, we seek to enhance traditional methodologies, demonstrating how quantitative analysis can provide valuable insights into construction techniques, proportions, and underlying design logic. First results highlight the potential of digital tools in broadening the understanding of cultural heritage, fostering new perspectives in architectural research and conservation.
This methodological shift is made possible by the convergence of two major technological and infrastructural trends.
The first is the growing number of Tangible Cultural Heritage (TCH) digitization projects being carried out internationally through large-scale 3D data capturing techniques. Continuous advancements in laser scanning technologies and photogrammetry algorithms are contributing to the democratization of 3D acquisition methods, leading to the widespread use of 3D data as a tool for documentation, conservation, and maintenance.
The second trend, which is more recent, concerns data management solutions for analysis. A significant number of cloud-based data-sharing initiatives—such as the 'ECHOES' project (1) and 'Open Heritage 3D' (McAvoy et al., 2024)—are trying to address the growing need to create digital commons and promote an open science approach within the Cultural Heritage sector.
The development of new digital tools within these virtual environments—enabling geometric, proportional, and mathematical analysis of 3D data—has the potential to profoundly transform research practices in the study of historical architecture.
By shifting the focus from indirect, text-based interpretations to direct, data-driven analysis of the built object, this approach challenges the long-established one within the humanities. In this sense, the integration of algorithmic computation into heritage research represents more than a methodological innovation: it marks a paradigm shift in the production of knowledge—one in which the monument itself becomes both the subject and the source of inquiry, and where interpretation emerges from measurable spatial information rather than from secondary narratives alone. In this perspective, having the possibility to access and analyses 3D digital versions of thousands of CH objects might enhance their understanding and interdisciplinary collaboration at international level.

1.1 Objectives

This research aims to test best-fit algorithms on point clouds of three Renaissance architectural masterpieces designed by Donato Bramante (1444–1514). The study will detail how the algorithms were applied to each case study and will discuss the "anomalies" identified. Furthermore, it seeks to provide a comprehensive analysis of the three cases, using statistical methods to highlight possible insights into the evolution of the architect's style during his Roman period.
To achieve this objective, the study is structured into four main phases. The first focuses on the identification of mathematical models and best-fit algorithms for analysing the column shafts. The second describes the point cloud features and outlines all pre-processing operations performed on the datasets. The third presents the case studies within the framework of their architectural significance. Finally, the fourth phase involves applying best-fit algorithms to the three case studies, presenting both individual and aggregated results.

(1) The ECHOES initiative brings together all the main actors of the European CH community to create a digital environment empowering users to interact with, manipulate and enrich Digital Twins. (https://www.echoes-eccch.eu/ visited on June 14th 2025)

Beyond the specific focus on the selected case studies, the research also aims to test and validate a workflow intended to expand the toolkit available for critical historical studies. By merging advanced computational methods with traditional analytical approaches, the study supports the interpretation of heritage sites and artifacts, enriching scholarly research. This integration of computational and traditional techniques enables new forms of analysis, ultimately enhancing the knowledge, preservation, and valorisation of cultural heritage.

2. Methodology

2.1 Mathematical models

The best-fitting models rely on regression techniques that minimize a function expressed as a curve, serving as a model to provide the closest approximation to a given dataset. In this experiment, a truncated prolate ellipsoid appears to be the most suitable shape for approximating a column shaft (Figure 1). A type of algebraic linear fitting in the parameters was employed, allowing for an analytical solution without the need for iterative methods. The function of our quadric model to be minimized (a truncated prolate ellipsoid in the case of a column shaft) can be expressed as (1),

$$\varphi(\mathrm{A,B,C,D,E,F,G,H,I}) = \sum_{i=1}^{n}\left(Ax_i^2 + By_i^2 + Cz_i^2 + Dy_iz_i + Ex_iz_i + Fx_iy_i + Gz_i + Hy_i + Ix_i + 1\right)^2 \quad (1)$$

where all the simple linear parameters can describe the characteristics of every possible quadric in space. A prolate ellipsoid is obtained by rotating an ellipse around its major axis. It is characterized by two equal equatorial semi-axes and a major axis around which the rotation occurs. Since this shape has minimal curvature at the base and maximum curvature at the apex, it has been considered the best approximation of the phenomenon of spinal tapering, given that the upper third is canonically regarded as "straight" compared to the remaining two-thirds, which tend to narrow.

Although geometric fitting generally provides higher accuracy compared to algebraic methods (Chernov, 2010), the complexity of the geometric model necessitates a balance between accuracy, computational efficiency, and overall computational complexity, depending on the application context.

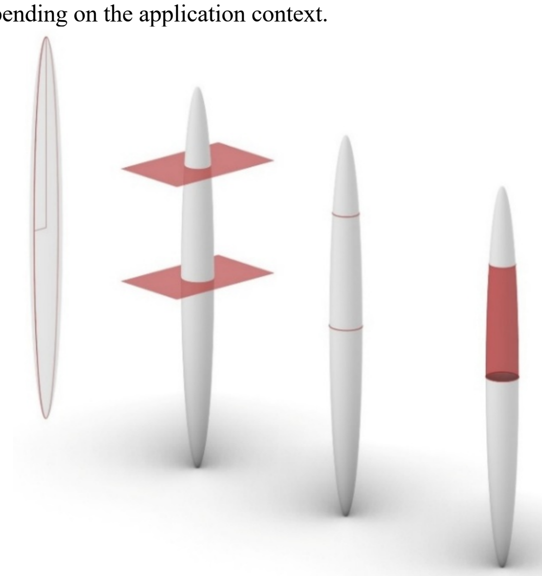

Figure 1. The construction of the column shaft as a portion of the ellipsoid prolate. (elaboration by authors)

We can compare the behavior of our ellipsoid fit to that of a circular one: both aim to minimize distances from a central element, whether the center coordinates for circumferences or the major axes of rotation for prolate ellipsoids. Chernov demonstrated that the algebraic fit (Kåsa method) closely approximates the geometric fit when data points are sampled along a full circle. However, when only a semicircle is sampled, the algebraic fit yields a slightly smaller circle than the geometric one. On smaller arcs, the algebraic method becomes nearly ineffective (Fig. 2, 3).

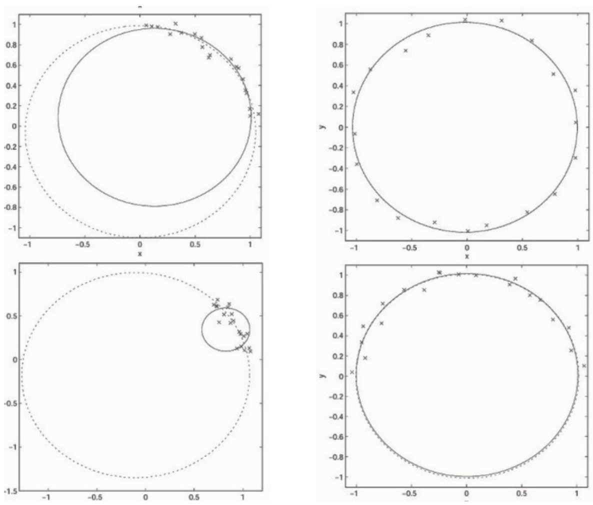

Figure 2. Four samples, each consisting of 20 points, were taken along different circular arcs of the circle $x^2+y^2=1$. The Kása fit (solid circle) demonstrates a clear underestimation of the radius as the arc length decreases. Gaussian noise of σ=0.05 was added to the data. (Adapted from Chernov, 2010, p. 106).

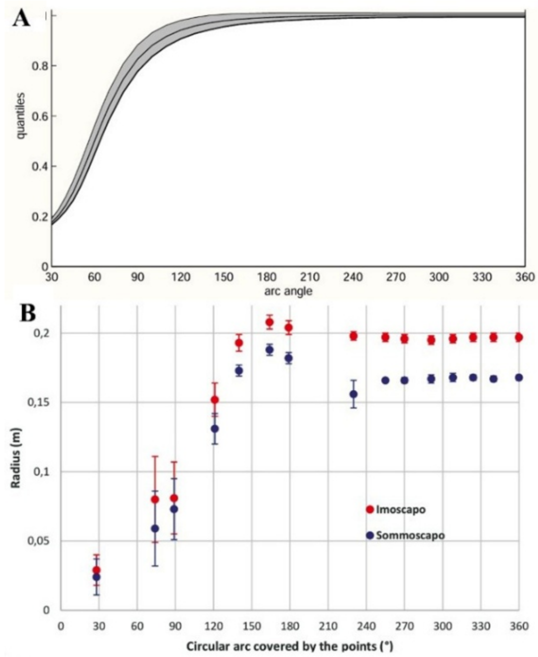

Figure 3. A) Radius estimates from Kása fit for points along circular arcs, showing systematic underestimation with shorter arcs (cf. Fig. 2). B) Best-fit algorithm performance for radii vs. arc coverage (tested on a Tempietto di San Pietro column). Both methods show comparable radius estimation dependence on point distribution, similar to the 2D circle fitting. (elaboration by the authors)

In summary, an algebraic algorithm performs exceptionally well when data points are sampled along a full circle or at least a semicircle. It also remains reliable when the points lie very close to a circular path, for instance if their deviations are significantly smaller than the instrument's margin of error. This bias in point distribution along the circumference not only leads to an underestimation of the radius but also affects the estimated center position on the plane. Extending this concept to the 3D space where paraboloids are considered, a discontinuous distribution of points over the surface would likely introduce similar biases in the computed center of gravity of the shaft.
The computational analysis was conducted in MATLAB, where the point cloud coordinates were imported and processed using custom scripts.

2.2 Best fit algorithms application

Once the parameters of the quadric are determined, the direction vector of the shaft’s rotation axis can be identified, along with its inclination relative to the vertical. By examining the distances of points from this axis in relation to the shaft’s height, the *imoscapo* and *sommoscapo radii*, as well as the total shaft height, can be calculated. These three dimensions lead to the definition of two additional derived parameters: the ratio between *imoscapo* and *sommoscapo*, and the ratio between the shaft’s height and the *imoscapo*. According to architectural treatises, these ratios establish the characteristic proportions of the different architectural orders (Migliari, 1991). Approximating the profile of a column shaft by an elliptical arc and analyzing its eccentricity facilitates the verification of the compatibility of the shaft’s tapering with other columns.
Furthermore, by calculating the spatial positions of the column shaft centers, the positional relationships between the various elements can be defined. With these dimensional and positional values established, a comprehensive analysis can be conducted evaluating the units of measurement employed in the architectural elements’ construction, examining their geometric relations, and assessing whether any observed discontinuities or deviations reflect intentional design decisions or construction practices.

2.3 Bramante in Roma, documentation and general overview on case studies

This research focuses on three masterpieces designed and built by Donato Bramante in Rome. Currently, these three architectural works— the cloister of Santa Maria della Pace, the staircase at the Vatican and the Tempietto of San Pietro in Montorio (Figure 4) —can be considered a manifesto of Renaissance architecture due to their deep connection with classical Roman language and the innovative contributions they each make to architectural history, particularly in terms of language, proportion and harmony (Bruschi, 2010).
Within this context, the use of architectural order plays a key role in Bramante’s architecture as a mean of expressing a distinct language through proportions and formal solutions.
Building on this premise, the experiment presented here focuses on analyzing the column shafts of these three structures. This fundamental architectural element represents the culmination of specific design principles combined with complex construction techniques for precise shape control.

More generally, from Ancient Greek and Roman period to the 19th century, the architectural order has served as a stone-bound reflection of evolving architectural cultures, ‘the core of mnemonic system by which forms were transmitted’ (Jones, 2009). During the Humanist period, the methods for measuring, designing, and proportioning architectural orders were conveyed through numerous treatises (2). These texts became points of reference for contemporary architects and remain essential today for understanding how classical orders were interpreted, as well as for accessing the system of rules and codes made explicit through Renaissance built architecture.

In this framework, through best fit algorithms we have the possibility to extract and interpret shafts construction rules regarding, for example, the profile of the entasis along the column or the ratio between *imoscapo* and *sommoscapo*. On this basis, it is possible to compare the column shafts of each case study with one another and, additionally, to compare the general rules employed in each monument with those described in the treatises as well as with those extracted from the analysis of classic buildings.

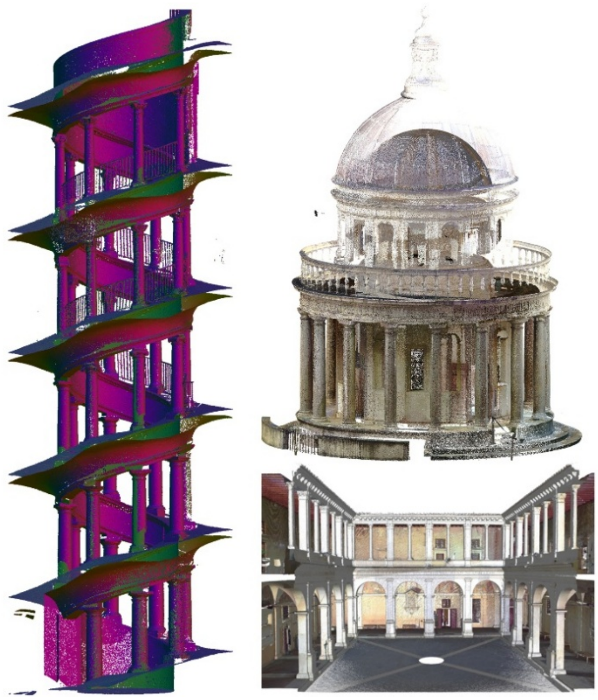

Figure 4. The three case studies: on the left, the staircase at the Vatican; on the top right, the Tempietto of San Pietro in Montorio, on the bottom right, the cloister of Santa Maria della Pace. (image by the authors).

The analysis has been conducted using as a base dataset the 3D laser scanner point clouds that have been pre-processed to ensure the points density homogeneity and filter the outliers.
The experiment considers only laser scanning point cloud because of the substantial homogeneity of shaft’s surfaces that would result in less morphological accuracy in photogrammetric reconstruction. Additionally, in terms of quality, laser scanner point clouds can be evaluated in a more structured way concerning both instrumental and alignment error.

(2) Because of their original contribution to the description and representation of architectural orders, it is worthy to mention: Leon Battista Alberti, *De re aedificatoria* (1452); Francesco di Giorgio Martini, *Trattato di Architettura civile e militare* (1478-1490), Sebastiano Serlio, *I Sette libri dell'architettura* (1537); Jacopo Barozzi Vignola, *Regola delli cinque ordini d'architettura* (1562); Andrea Palladio, *I quattro libri dell'architettura* (1570).

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Firstly, point clouds have been segmented and classified to isolate column shafts from the base and the capitals, then the S.O.R filter was used to remove outliers and finally the point cloud was decimated to ensure density homogeneity.
In this regard, we decided to use the minimum value of density calculated for each shaft as the reference value for point cloud decimation. This operation was needed because the point cloud density variations might affect negatively the accuracy of the best fist geometry reconstruction as discussed in the previous paragraph. Tests on shaft point clouds demonstrate that when points are properly distributed across the entire surface, decimating the original point cloud does not significantly affect the best-fit results, either in terms of dimensional accuracy or positional values except for besides minor, expected fluctuations in the output values (Figure 5).

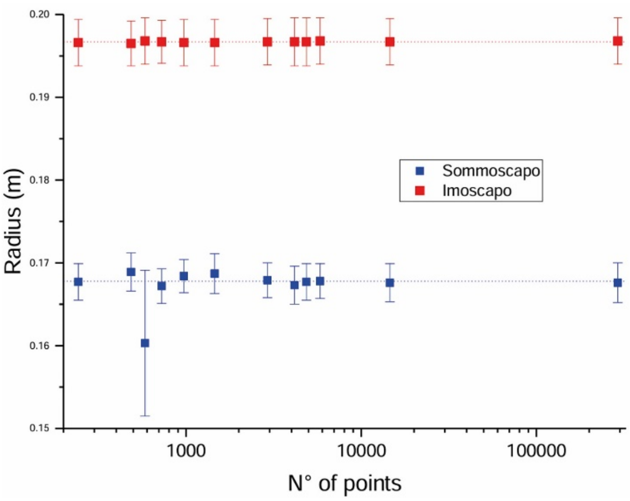

Figure 5. the radii returned by the best-fit algorithm, demonstrates that as long as the points are uniformly distributed along the shaft's surface, a spatial decimation of the points can be performed while still providing robust results (test performed on a column from the Tempietto). (image by the authors).

3. The Santa Maria della Pace Cloister

3.1 Historical and architectural context

The cloister of Santa Maria della Pace is the first building designed by Bramante after his arrival in Rome in 1499 (Bruschi, 2010). It is conceived to contain in just two storeys the four architectural orders, the Tuscanic of the ground floor pillars, the Ionic of ground floor parasters, the Composite of the first floor pillars and the Corinthian of the columns at the first floor. Starting in 2019, a 3D documentation campaign was launched to support the documentation and management of the complex through digital models (3). Following well-established methodologies, an integrated capturing campaign was carried out: 3D laser scanning was applied to all the main areas of the cloister and museum halls; drone-based photogrammetry enabled the survey of the roof and inner façades of the cloister; and a topographic survey served as the main reference system to optimize photogrammetric processes and to align the resulting point clouds with those generated by laser scanning.

This research focused on the 16 Corinthian columns located on the four sides of the cloister. The goal is to verify whether the proportions between *sommoscapo* and *imoscapo*, as well as the ones between the diameter and the height, might suggest a specific code used by the architect. Additionally, the columns have been compared each other to verify their formal consistency and identify major differences.

3.2 Best fit algorithm application

Best-fit analyses were performed on the sixteen column shafts for dimensional and spatial characterization, including an estimation of the polygon (square) side lengths formed by their centroids and the intercolumnar spacing. The results are summarized in the Table 1.

Imo. (m)	Som. (m)	H (m)	$Ratio_{I/S}$
0.296±0.005	0.249±0.003	2.592±0.011	1.192±0.033
S_{fit} (m)	e		D (m)
14.106±0.020	0.999479±0.000020		3.509±0.009

Table 1. Imo.=mean diameter (Imoscapo), Som.= mean diameter (Sommoscapo), H= mean of the shafts, RatioI/S=ratio between Imo. and Som., Sfit= Mean side length of the best-fit square, e= Mean eccentricity of the ellipse fitted to the shaft profile, D=mean Inter-centroid distances.

3.3 Critical analysis

The *imoscapo* and *sommoscapo* diameters show slight variations (approximately of 2 cm) depending on the column's position within the cloister. As shown in Figure 6, the minimum shaft diameters are found at opposite corners (columns 1 and 8), while the maximum values occur at the diagonally opposite side (columns 4 and 13). This distribution may be related to a perspectival strategy applied to the cloister, particularly when viewed from its entrance, located precisely at one of these vertices. This periodic trend, with peaks and minimum in diameter measurements, also extends to the cloister's sides and is clearly visible in Figure 6. The ratio between *imoscapo* and *sommoscapo* is close to 6/5, and the *imoscapo* itself matches the roman foot of 0.296 m. The shaft heights remain nearly constant, equivalent to 8¾ times the *imoscapo* length. Regarding the polygon formed by interpolating the column centroids, it essentially defines a square with sides measuring 14.106m, approximately four times the average distance between centroids. Notably, the smallest intercolumniations occur along the wall containing columns 13, 14, 15, 16 (Figure 7).

(3) The documentation and representation of the Bramante Cloister was completed within the framework of the research project “Amen. Augmented Museum Environment Network”. Funding: DTC Lazio; Lead Partner: DART S.r.l./ Centro Culturale Internazionale S.r.l. – Chiostro del Bramante; Partners: Fondazione Mario & Maria Pia Serpone, Segni d’Arte, Sapienza University of Rome, Department of History, Representation and Restoration of Architecture (DSDRA)

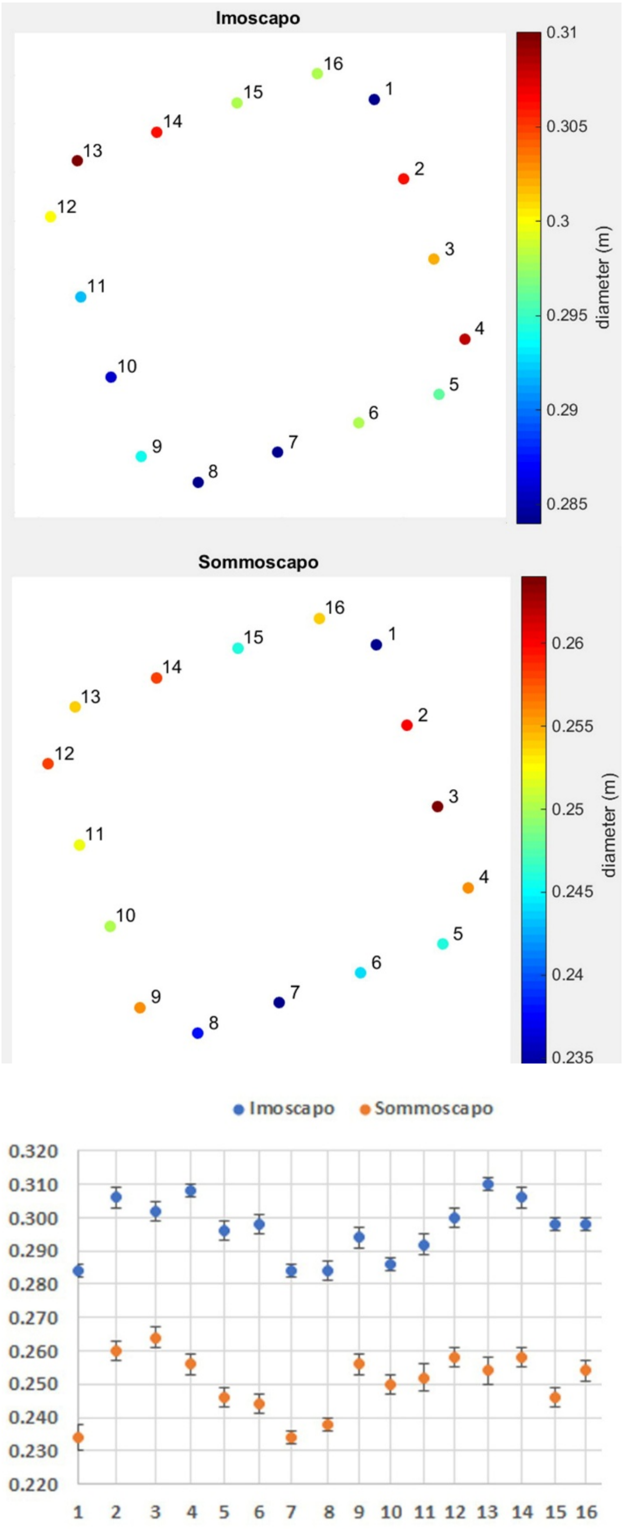

Figure 6. Diameter trends of imoscapo and sommoscapo in Santa Maria della Pace cloister. Top, Spatial distribution of diameter values across column positions. Bottom, periodic variation pattern showing minima and maxima aligned with the cloister's diagonals. (image by the authors).

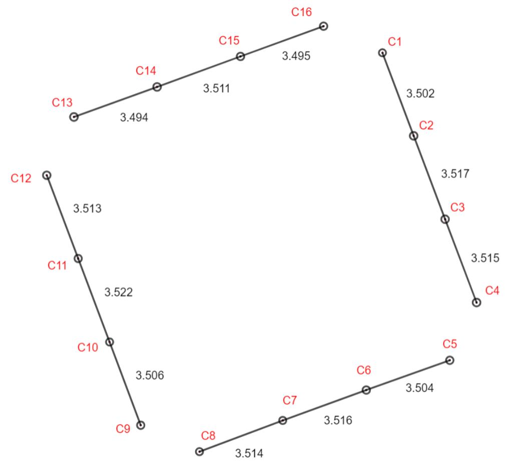

Figure 7. centroid distances in the cloister. The side containing columns 13-16 shows the smallest intercolumnio values. (image by the authors).

4. The Vatican staircase

4.1 Historical and architectural context

The staircase at the Vatican Museum is conceived as a void cylinder with a spiral entablature sustained by columns. It allows the access to the sculptures' courtyard from the countryside (Bruschi, 2010). The staircase construction started probably in 1507 and was not completed yet at Bramante's death in 1514. Bramante decide to design the supports of the entablature using classical orders in sequence: the tuscanic, the doric, the ionic and the composite. Given the spatial continuity of the staircase, this choice results in having at each level two columns, one next to each other, of two different architectural orders and with different proportions.

In 2020, the staircase was surveyed with a 3D laser scanner to document how classical order sequence was constructed and investigate spatial relationship between the five different levels.

Regarding the application of best fit algorithms, the research aimed to study the changing proportions of column shaft for each architectural order to possibly identify design illusionistic solutions and how the changing between two consecutive architectural orders have been solved.

4.2 Best fit algorithm application

Best-fit analyses were conducted on the column shafts to assess their dimensional and spatial properties. Additionally, a helical fit was performed to estimate the centroid around which the staircase spiral revolves. In this case study, which involves 36 columns, each with unique diameter, ratio, and eccentricity values, the data are shown directly in scatter plots rather than a table.

4.3 Critical analysis

The radii of the column shafts exhibit a gradual decrease as the staircase ascends and spirals upon itself. Due to structural requirements, their heights remain constant throughout the ascent, creating a perceptual effect in which the ratio between shaft height and column diameter diminishes toward the top of the staircase (Figure 8).

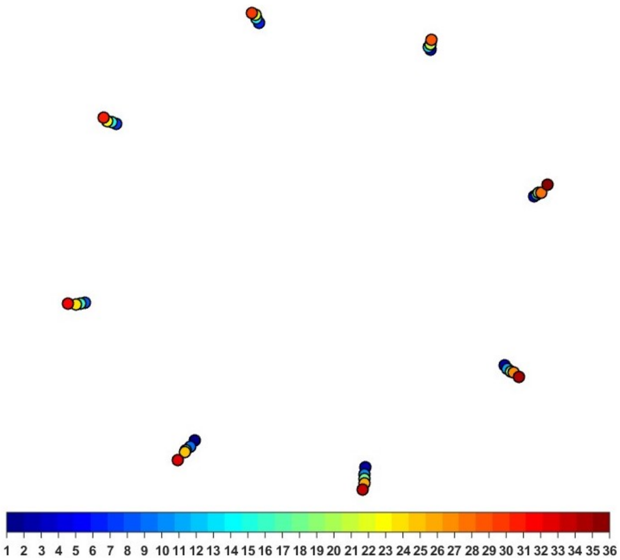

Figure 8. Planar projection (XY plane) of the centroids in Bramante's staircase. The centroids exhibit centrifugal displacement as they ascend, with their divergence pattern directly influenced by their position relative to the columnar arrangement of the structure. (image by the authors).

By plotting the *imoscapo* and *sommoscapo* values for the nine columns in each architectural order, a proportional logic emerges, linking the trends in their radii. When the 36 columns are divided into four groups corresponding to the four distinct architectural orders, the *imoscapo* values at position 9 of each order align closely with the *sommoscapo* values at position 1 of the subsequent order. If Bramante applied this logic (within reasonable tolerance margins), it is possible that he followed a mathematical progression to establish a consistent rule. Furthermore, when projecting all column shafts onto the XY plane, a spatial distribution pattern becomes evident: as the staircase rises, the center of each column shifts outward in a direction that could be described as centripetal. In three-dimensional space, this arrangement generates a helical motion that progressively widens with increasing elevation, reinforcing the dynamic interplay between form and structural necessity (Figure 9).

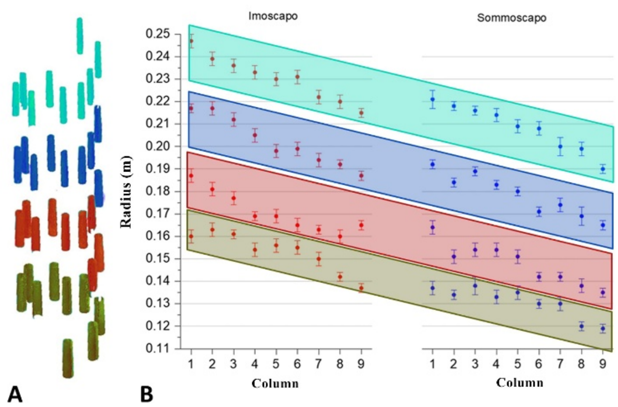

Figure 9. Classification of the 36 columns into four progressive orders along Bramante's staircase height. B) Trend of imoscapo and sommoscapo values across the four orders, with colors indicating correspondence to each architectural order. (image by the authors)

5. The Tempietto of San Pietro in Montorio

5.1 Historical and architectural context

The Tempietto of San Pietro in Montorio is the second monument that Bramante design and builds after his arrival in Roma. Most probably, the construction started between 1503 and 1505 (Cantatore, 2016) and just after its completion was already conceived as a prominent example of architecture by Bramante's contemporaries. The building is conceived as a circular temple articulated in three levels: the crypt, the cylindric cella surrounded by 16 doric columns and the drum covered by a dome. The composition of the different elements and the main proportions of the building are coherent with the description of circular peripteral temples that Vitruvius suggests in his *De Architectura* (book IV, cap. 8), while concerning the unit of measure used to dimension the module and size the elements, some hypothesis have been made converting the length of the module (the diameter of the column at the *imoscapo*) according to the unit of measure in use in Milan and in Rome (Bianchini, 2017). The first 3D point cloud of the tempietto was elaborated in 2013 and is the result of an integrated survey based on both topographic and laser scanning data. Since then, additional survey campaigns were conducted to update the original data (4).

5.2 Best fit algorithm application

Best-fit analyses were conducted on the sixteen column shafts to assess their dimensional and spatial properties, as well as a circular best-fit on their centroids. The results are presented in the table 2.

Imo. (m)	**Som. (m)**	**H (m)**	**Ratio$_{I/S}$**
0.398±0.005	0.349±0.003	2.957±0.012	1.140±0.017
R$_{fit}$ (m)	**e**		
3.954±0.002	0.9993872±0.0000239		

Table 2. Imo.=mean diameter (Imoscapo), Som.= mean diameter (Sommoscapo), H= mean of the shafts, RatioI/S=ratio between Imo. and Som., Rfit=radius of best-fit circle to shafts' centroids, e= Mean eccentricity of the ellipse fitted to the shaft profile.

5.3 Critical analysis

As shown in the table 2, the 16 columns exhibit relatively consistent values for the *imoscapo* and *sommoscapo*, as well as in the shaft heights. Regarding the proportions between architectural elements and metric values, the *sommoscapo* and *imoscapo* maintain a ratio of approximately 7/8, while the shaft height equals roughly 10 x 3/4 of the *imoscapo*. It is also worth noting that the shaft height corresponds to 10 times a foot unit of 0.2978 meters. By calculating the circumference that best approximates the centroid of the columns on the floor, we obtain a radius equal to 10 times the average *imoscapo* value. These measurements align with those found in previous studies investigating the modular system of the Tempietto (Bianchini, 2017).
By drawing hypothetical lines connecting the centres of the columns to the centre of this derived circumference, we were able to analyse the angular spacing between them (Figure 10). Ideally, with 16 columns, one would expect a uniform 22.5° division for each sector. However, a slight deviation of 0.4° (equivalent to about 3 cm along the final arc) was detected in the sector corresponding to the main entrance.

(4) The last data captiring activities were carried out in 2024 by the Department of History, Representaion and Restoration of Architecture as part of the doctoral research of Salvatore Di Pace, titled *'Architecture and Space: Communicating through Sensory Substitution Devices'* (supervisors: Professors Alfonso Ippolito and Francisco Juan-Vidal).

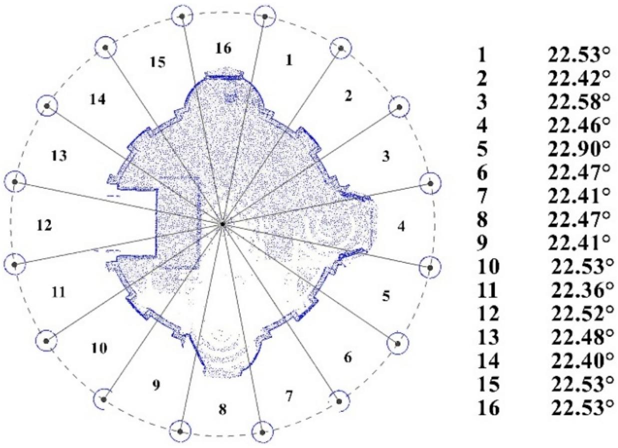

Figure 10. angular distribution of columns relative to the center in San Pietro e Montorio. Sector 5 exhibits the widest angular divergence. (image by the authors).

6. Cross analysis on case studies

Finding a proper statistical and graphical method to compare the three case studies of Bramante in order to identify possible compatibilities or relationships between them is not an easy task. This is further complicated by the fact that, in the case of Bramante's staircase, we deal with 36 columns, each with different diameter values. To facilitate a clear visual interpretation, we chose to group these into four distinct samples, each corresponding to a specific architectural order. As shown in Figure 11, which correlates the ratios between shaft height and *imoscapo* as well as shaft eccentricity of the columns, the three case studies form three separate clusters. We also included two additional samples from classical architecture: the *Pantheon* and the *Temple of Antoninus and Faustina*. Bramante's case studies, along with these two classical examples, deviate from the proportional criteria described by architectural treatises for column elements across different orders. It is also worth noting that the two ancient examples exhibit very close values, while the Tempietto of San Pietro in Montorio does not deviate significantly from them in statistical term

7. Conclusions

This research aims to translate the study of architectural language into a scientifically grounded approach by applying statistical models to support interpretation. The results presented here can be considered a preliminary basis for further studies on Bramante's architecture.

Rather than offering definitive answers, this methodology provides an additional perspective from which to examine the monuments, allowing for direct interrogation of the built artifact. It is worth noting that best-fit algorithms can support diachronic analysis and help identify recurring rules across buildings from different periods and contexts. This capability can reveal connections between architects and architectural works, suggesting possible sources of inspiration and echoes through time. This is exemplified by the cross-analysis presented in this study. We know that after arriving in Rome, Bramante studied and measured several classical buildings available to him at the time, and the Pantheon was likely among them. Within Bramante's body of work, the Pantheon is known to have served as a model for the Tempietto of San Pietro in Montorio, both in terms of modular proportions and architectural composition (5). Thanks to this research, we can now make this close relationship explicit and visible in terms of the columns' proportions and shapes, based on accurate statistical analyses. In contrast, the shafts of the Cloister of Santa Maria della Pace do not exhibit significant consistency with classical models. This may be related to the fact that, as previously mentioned, Bramante designed the cloister shortly after his arrival in Rome. For this reason, it can be assumed that his references at the time were still linked to those circulating in Milan, the city where he had worked prior to moving to Rome.

Furthermore, this research enabled us to focus not only on individual elements, but also to evaluate the rhythm of the shafts and their role in expressing perceptual dynamism through architecture. This is particularly evident in the case of the Vatican staircase, where the sinusoidal trend in the *sommoscapo/imoscapo* ratio can be seen as meaningful in decoding Bramante's design principles.

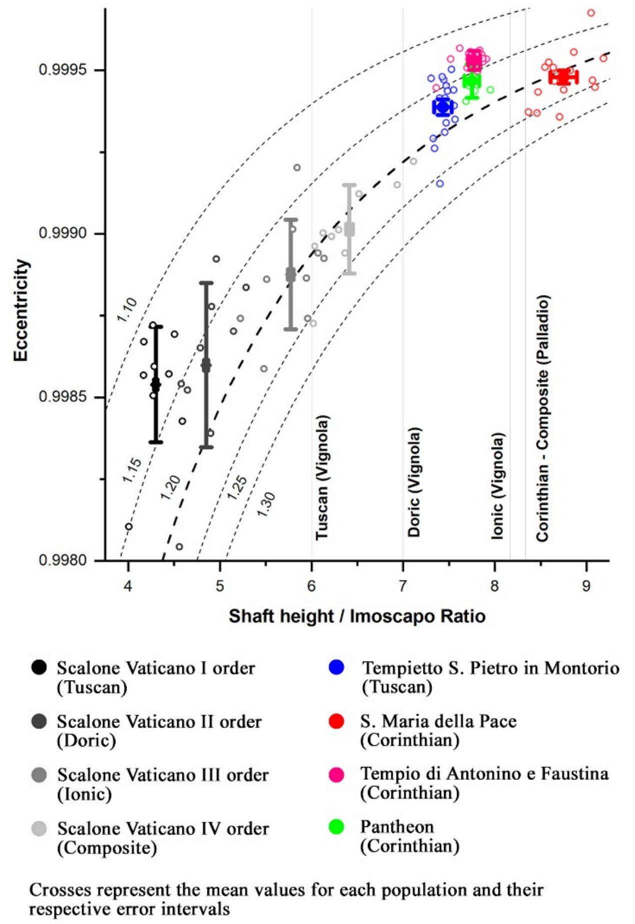

Figure 11 Distribution of the analyzed column populations. The discrepancy between the observed proportions of the architectural orders and those described by the treatises is evident. The dotted lines represent the Imoscapo/Sommoscapo ratio. (image by the authors).

(5) Starting from Serlio S. (1540) *Il terzo libro di Sebastiano Serlio bolognese* p. XLII, to the more recent bibliography: Marías F. (2017), Los clientes del Tempietto: historia, intenciones y significados. In F. Cantatore (Ed.), *Il Tempietto di Bramante nel monastero di San Pietro in Montorio*, pp. 111-152. Roma: Edizioni Quasar

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