COMPARISON AND ANALYSIS OF NONLINEAR LEAST SQUARES METHODS FOR VISION BASED NAVIGATION (VBN) ALGORITHMS

B. Sheta [a, *], M. Elhabiby [a, b], N. El-Sheimy [a]

[a] Dept. of Geomatics, University of Calgary, Calgary, Alberta, T2N 1N4 - (bimsheta, mmelhabi, elsheimy)@ucalgary.ca
[b] Public Works Department, Faculty of Engineering, Ain Shams University, Cairo, Egypt -mmelhabi@ucalgary.ca

Commission ICWG I/V

KEY WORDS: UAV, Nonlinear Optimization, Vision Based Navigation

ABSTRACT:

A robust scale and rotation invariant image matching algorithm is vital for the Visual Based Navigation (VBN) of aerial vehicles, where matches between an existing geo-referenced database images and the real-time captured images are used to georeference (i.e. six transformation parameters - three rotation and three translation) the real-time captured image from the UAV through the collinearity equations. The georeferencing information is then used in aiding the INS integration Kalman filter as Coordinate UPdaTe (CUPT). It is critical for the collinearity equations to use the proper optimization algorithm to ensure accurate and fast convergence for georeferencing parameters with the minimum required conjugate points necessary for convergence. Fast convergence to a global minimum will require non-linear approach to overcome the high degree of non-linearity that will exist in case of having large oblique images (i.e. large rotation angles).The main objective of this paper is investigating the estimation of the georeferencing parameters necessary for VBN of aerial vehicles in case of having large values of the rotational angles, which will lead to non-linearity of the estimation model. In this case, traditional least squares approaches will fail to estimate the georeferencing parameters, because of the expected non-linearity of the mathematical model. Five different nonlinear least squares methods are presented for estimating the transformation parameters. Four gradient based nonlinear least squares methods (Trust region, Trust region dogleg algorithm, Levenberg-Marquardt, and Quasi-Newton line search method) and one non-gradient method (Nelder-Mead simplex direct search) is employed for the six transformation parameters estimation process. The research was done on simulated data and the results showed that the Nelder-Mead method has failed because of its dependency on the objective function without any derivative information. Although, the tested gradient methods succeeded in converging to the relative optimal solution of the georeferencing parameters. In trust region methods, the number of iterations was more than Levenberg-Marquardt because of the necessity for evaluating the local minimum to ensure if it is the global one or not in each iteration step. As for the Levenberg-Marquardt method, which is considered as a modified Gauss-Newton algorithm employing the trust region approach where a scalar is introduced to assess the choice of the magnitude and the direction of the descent. This scalar determines whether the Gauss-Newton method direction or the steepest descent method direction will be used as an adaptive approach for both linear and non-linear mathematical models and it successfully converged and achieved the relative optimum solution. These five methods results are compared explicitly to the linear traditional least-squares approach, with detailed statistical analysis of the results, with emphasis on the UAV (VBN) applications.

1. INTRODUCTION

In recent years, the utility of the Unmanned Aerial Vehicles (UAV) has greatly increased in applications such as, surveillance, law enforcement and aerial mapping. Furthermore, UAVs are quickly becoming an integral part of both military and commercial operations. For these classes of applications, accurate UAV navigation is considered a critical element to be investigated. Inertial Navigation Systems (INS) with Inertial Measurement Unit (IMU) and Global Positioning System (GPS) are frequently used in airborne applications.

On one hand, the INS systems are passive systems, which are widely used in navigation and characterized by its immunity against jamming. On the other hand, the GPS aided system is commonly used to correct the navigation error drift caused by using INS alone. However, a GPS aided system has some limitations. When using GPS, the navigation system is dependent upon the availability of satellites necessary for the operation and how reliable the navigation system can be in making decisions about the information received from these satellites. Examples of such situations are the jamming of GPS signal leading the navigation system to not rely on such a signal or, urban areas where loss of satellites is expected (Nilsson 2005).

To achieve robust UAV navigation in such environments, alternative navigation system is employed, such as Vision based Navigation (VBN), where inertial sensors are employed together with low cost imaging sensors which make measurements of the surrounding environment in order to provide the necessary information required for navigation. The main objective of this paper is investigating the estimation of the georeferencing parameters necessary for VBN of aerial vehicles in case of having large values of the rotational angles, which will lead to non-linearity of the estimation model. In this case, traditional least squares approaches will fail to estimate the georeferencing parameters, because of the expected non-linearity of the mathematical model.

* Corresponding author. This is useful to know for communication with the appropriate person in cases with more than one author.

Five different nonlinear least squares methods are presented for estimating the transformation parameters. Four gradient based nonlinear least squares methods (Trust region, Trust region dogleg algorithm, Levenberg-Marquardt, and Quasi-Newton line search method) and one non-gradient method (Nelder-Mead simplex direct search) is employed for the six transformation parameters estimation process.

2. COLLINEARITY EQUATION AND ITS APPLICATION TO VBN

In a VBN system, given conjugate matched points between the real-time images and the images in the database, the transformation parameters (ω, φ, κ, X_{PC}, Y_{PC}, Z_{PC}) can be estimated through the mathematical model of the collinearity equation, shown below, using the proper optimization algorithm to ensure accurate and fast convergence for those parameters with the minimum required conjugate points necessary for convergence.

$$x_p = x_{pc} - f\,\frac{c_{11}(X_P - X_{PC}) + c_{21}(Y_P - Y_{PC}) + c_{31}(Z_P - Z_{PC})}{c_{13}(X_P - X_{PC}) + c_{23}(Y_P - Y_{PC}) + c_{33}(Z_P - Z_{PC})}$$
$$y_p = y_{pc} - f\,\frac{c_{12}(X_P - X_{PC}) + c_{22}(Y_P - Y_{PC}) + c_{32}(Z_P - Z_{PC})}{c_{13}(X_P - X_{PC}) + c_{23}(Y_P - Y_{PC}) + c_{33}(Z_P - Z_{PC})} \tag{1}$$

where f, x_{pc}, y_{pc} = camera interior orientation parameters
x_p, y_p = image coordinates
X_{PC}, Y_{PC}, Z_{PC} = coordinates of projection center
X_P, Y_P, Z_P = object coordinates in ground coordinate system
$c_{11},\ldots,c_{33}$ = elements of the 3D rotation matrix parameters

The transformation parameters are estimated through an optimization algorithm using the measured common points in both the aerial image and referenced database(Saeedi 2009). Once completed, the camera position and attitude will be estimated and consequently the UAV pose can be estimated. There are two general approaches to investigate the nonlinear least squares algorithms(El-Habiby, Gao et al. 2009). The first one is linearization of the model. This approach is employed by adding corrections to the unknown estimated parameters during the iteration steps. However, in the general photography case without the near vertical assumption, the second approach is used. The second approach is based on gradient method, where modifications are made to the steepest descent approach.

3. TRANSPORMATION PARAMETERS ESTIMATION BASED ON NONLINEAR LEAST SQUARES

A pose estimation problem can be investigated through the correspondence between the 3D Ground Control Points (GCP) and the corresponding 2D points in image coordinates(Lu, Hager et al. 2000). Gauss-Newton and Levenberg-Marquardt, among other methods such as Trust Region method, are considered as classical iterative approaches for solving the non-linear least square problem for pose estimation problem(Lowe 1991).
In general, minimization equation for unconstrained nonlinear problem can be described as in equation (2):

$$\min F(x)\ ,\ x \in R^n, F : R^n \to R \tag{2}$$

where x = unknown parameters to be estimated (transformation parameters)
$F(x)$ = objective function (collinearity equation)

3.1 Nelder-Mead Simplex Direct Search Method

In this method, a simplex S of approximations to an optimal point is maintained. This is achieved by sorting the vertices x_i to x_{k+1} based on the objective function values.

$$f(x_1) \le f(x_2) \le \ldots \le f(x_{k+1}) \tag{3}$$

where x_1 = the best vertex
x_{k+1} = the worst vertex

Reflection (ρ), expansion (χ), contraction (γ), and shrinkage (σ) parameters are used in minimizing the objective function(Nelder and Mead 1965).

3.2 Trust Region Method

In this method, a quadratic model is described by equation (4)(Andrew R. Conn 2000):

$$m_c(x) = f(x_c) + \nabla f(x_c)^T (x - x_c) + (x - x_c)^T H (x - x_c)/2 \tag{4}$$

where $f(x_c)$ = objective function at current iteration
H = model Hessian

At each iteration x_c, trust region radius Δ is computed using the following equation:

$$\beta = \left\{ x \in R^n\ , \|x - x_c\| \le \Delta_c \right\} \tag{5}$$

where Δ_c = trust region radius

Starting with the quadratic model m_c and the trust region radius Δ, a trial point $x_c + s_t$ is calculated such that the model is minimized. Then, the objective function is computed at the trial point and compared to the quadratic model value at the trial point.

3.3 Trust Region Dogleg Method

In this method, the search direction is computed over a path $s(\alpha)$ where $0 \le \alpha \le \alpha_{max}$ such that the model mc is minimized over the path $s(\alpha)$. The solution of the trust region problem lies on such path minimizing the objective function. The path is given by the following equation(Kelley 1999):

$$s(\alpha) = \begin{cases} \alpha s^{cauchy} & 0 \le \alpha \le 1 \\ s^{cauchy} + (\alpha - 1)(s^{GN} - s^{cauchy}) & 1 \le \alpha \le 2 \end{cases} \tag{6}$$

where s^{cauchy} = Cauchy point
s^{GN} = Gauss-Newton Point

3.4 Levenberg-Marquadrt Method

This method is considered as a modification for the Gauss-Newton method to overcome the problem of the failure of such method when $J_c^T J_c$ is singular. Where, J_c is the jacobian of the objective function at the current point. To overcome this problem, a coefficient $\lambda>0$ is introduced such that the search direction can be found using the following equation(Luenberger 1984):

$$(J_c^T J_c + \lambda I) d_c = -J_c^T f(x_c) \tag{7}$$

where d_c = search direction

Depending on the value of λ, the Levenberg approach will either be a Gauss-Newton one (when $\lambda=0$) or steepest descent approach (when λ has big values).

3.5 Quasi-Newton Line Search Method

The basic idea behind this method is that approximation of the Hessian matrix H of the objective function is achieved by a positive definite matrix. This positive definite matrix is initially computed by H_0. As the search continues, the approximating matrix is updated at each iteration step and consequently the second derivative information is updated and improved. The Broyden, Fletcher, Goldfarb, and Shanno (BFGS) update is given by the following(BROYDEN 1970):

$$H_{k+1} = H_k - \frac{H_k s_k (H_k s_k)^T + \delta_k \gamma_k^T H_k}{s_k^T H_k s_k} + \frac{v_k v_k^T}{v_k^T s_k} \tag{8}$$

where $s_k = \alpha_k d_k$
$v_k = \nabla f(x_{k+1}) - \nabla f(x_k)$
α_k= step size.

4. PERFORMANCE ANALYSIS

Solution of the collinearity equation given in equation (1) is done using the proposed five nonlinear methods. Simulated data is used such that three situations are realized. The first one is the near vertical assumption, then a general tilted photograph is realized with φ=10° and 30°.The camera focal length is 151.876 mm. Three ground control points are used in the tests. Results shown below are the residuals of the proposed nonlinear least squares methods and the number of iterations achieved.

Table 1 Initial values for object space transformation parameters

	Ω (rad)	*Φ (rad)*	*K (rad)*	*X_0 (m)*	*Y_0 (m)*	*Z_0 (m)*
value	0	0	1.564	6134.5	22,851.1	1100

Table 2 Image points and ground control points coordinates

	Image coordinates (mm)		Ground Control Coordinates (m)		
	x	y	X	Y	Z
1	-53.845	65.23	6934.954	23,961.10	160.13
2	104.5	68.324	7860.202	23,941.56	152.65
3	-61.372	-79.559	6836.65	23,087.47	137.71

Table 3 Performance of nonlinear least squares methods with near vertical assumption

Object Space Parameters	Trust Region	Trust Region Dogleg	Levenberg-Marquadrt	Quasi-Newton Line Search	Nelder-Mead Simplex Search
$\delta\omega$	0.049	0.066	0.007	0.040	0.006
$\delta\varphi$	0.011	0.002	0.011	0.040	0.011
$\delta\kappa$	0.003	0.011	0.001	0.038	0.007
δX_0	20.406	16.834	16.703	16.833	18.569
δY_0	7.174	7.048	7.037	7.028	55.402
δZ_0	0.261	1.316	1.216	1.316	4.402
No. of Iteration	5	3	2	3	159

Table 4 Performance of nonlinear least squares methods with φ = 10°

Object Space Parameters	Trust Region	Trust Region Dogleg	Levenberg-Marquadrt	Quasi-Newton Line Search	Nelder-Mead Simplex Search
$\delta\omega$	0.048	0.037	0.021	0.035	0.006
$\delta\varphi$	0.021	0.108	0.628	0.040	0.138
$\delta\kappa$	0.007	0.053	0.544	0.027	0.029
δX_0	20.033	73.489	295.781	16.833	166.201
δY_0	5.572	13.570	350.136	5.048	18.158
δZ_0	1.151	29.256	420.242	1.302	30.572
No. of Iteration	4	4	50	3	155

Table 5 Performance of nonlinear least squares methods with φ = 30°

Object Space Parameters	Trust Region	Trust Region Dogleg	Levenberg-Marquadrt	Quasi-Newton Line Search	Nelder-Mead Simplex Search
$\delta\omega$	0.051	0.071	0.041	0.004	0.006
$\delta\varphi$	0.038	0.504	0.601	0.038	0.262
$\delta\kappa$	0.029	0.385	0.510	0.041	0.021
δX_0	28.672	320.11	300.214	16.803	298.142
δY_0	7.763	242.29	302.229	5.038	62.517
δZ_0	8.604	380.56	402.217	1.316	8.026
No. of Iteration	8	8	37	3	153

5. CONCLUSION

Object space transformation parameters estimation was introduced using five nonlinear least squares methods. These parameters are necessary for pose estimation of UAV. Two situations were introduced. The first one is the near vertical assumption and the second one is the general tilted photograph case. Convergence of the proposed nonlinear least squares methods were tested in the situations mentioned before. As for the Nelder-Mead simplex direct search method, it failed to estimate the object space transformation parameters because this method depends on the objective function only without any derivative information. As for the other four methods, the object space transformation parameters were successfully estimated for Φ = 0°. However, when Φ = 10° and 30°, the object space transformation parameters were successfully estimated only by Quasi-Newton line search method due to depending on the Hessian matrix of the Collinearity equation.

6. ACKNOWLEDGMENTS

This work was supported in part by research funds from TECTERRA Commercialization and Research Centre, the Canada Research Chairs Program, and the Natural Science and Engineering Research Council of Canada (NSERC).

7. REFERENCES

Andrew R. Conn, N. I. M. G., Philippe L. Toint (2000). Trust Region Methods, Society for Industrial and Applied Mathematics.

BROYDEN, C. G. (1970). "The Convergence of a Class of Double-rank Minimization Algorithms 1. General Considerations." IMA Journal of Applied Mathematics **6**(1): 76-90.

El-Habiby, M. M., Y. Gao, et al. (2009). "Comparison and Analysis of Non-Linear Least Squares Methods for 3-D Coordinates Transformation." Survey Review **41**(311): 26-43.

Kelley, C. T. (1999). Iterative Methods for Optimization, Society for Industrial and Applied Mathematics.

Lowe, D. G. (1991). "Fitting parameterized three-dimensional models to images." Pattern Analysis and Machine Intelligence, IEEE Transactions on **13**(5): 441-450.

Lu, C. P., G. D. Hager, et al. (2000). "Fast and globally convergent pose estimation from video images." Ieee Transactions on Pattern Analysis and Machine Intelligence **22**(6): 610-622.

Luenberger, D. G. (1984). Linear and nonlinear programming. Reading, Mass., Addison-Wesley.

Nelder, J. A. and R. Mead (1965). "A SIMPLEX-METHOD FOR FUNCTION MINIMIZATION." Computer Journal **7**(4): 308-313.

Nilsson, J. (2005). Visual Landmark Selection and Recognition for Autonomous Unmanned Aerial Vehicle Navigation. Department of Numerical Analysis and Computer Science. Stockholm, Sweden, Royal Institute of Technology. **Master's Degree Project:** 46.

Saeedi, S. S., Farhad; El-Sheimy, Naser. (2009). "Vision-Aided Inertial Navigation for Pose Estimation of Aerial Vehicles." Proceedings of the 22nd International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS 2009): 453-459.